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ABSTRACT

This paper gives results of tests carried out on reinforced concrete beams in the establishment of a basis for design of concrete ships during the World War.

Most of the beams were of I-shaped cross section. The web thickness varied from 2 to 12 inches, the depth from 18 inches to 10 feet, and the span from 9 feet 6 inches to 20 feet.

The web reinforcement generally consisted of loose stirrups placed vertically, or inclined at 45°. In a few beams expanded metal, and in a few others horizontal bars in the web, were used as reinforcement.

Cracks generally began to appear in the beams at shearing stresses of 100 to 300 lbs./in.^{2}. Previous to the formation of cracks there was generally very little stress in the web reinforcement.

The tensile stress in the web and the shearing strength of the beam were generally independent of the compressive strength of the concrete of the beam and directly dependent upon the amount of web reinforcement.

As measured by ultimate shearing strength, vertical and inclined stirrups were about equally effective, pound for pound of steel, in reinforcing the web to resist shear. The inclined stirrups were more effective than the vertical stirrups in preventing deflection and in resisting the widening of cracks. The statements of this paragraph refer to beams having only vertical or only inclined stirrups. Where both kinds of stirrups were present in the same beam the inclined stirrups took nearly twice as great stress as the vertical stirrups until the yield point was reached. After the yield point was reached the stress in the vertical stirrups became nearly equal to that in the inclined reinforcement, and the maximum load carried was about the same as that for a beam having the same total amount of web reinforcement, but all of one type, either vertical or inclined.

The shearing strengths found were generally much higher than have been obtained in previous investigations. This probably is due mainly to the use of larger quantities of web reinforcement, combined with sufficient anchorage of stirrups and longitudinal bars to permit the stirrups to be effective.

CONTENTS

I. INTRODUCTION

This paper gives results of tests on reinforced concrete beams (series 1, 4, and 10) made by the concrete ship section of the Emer-

gency Fleet Corporation and the United States Bureau of Standards. The investigation consisted principally of two series of tests, one of which was carried out at the Pittsburgh Laboratory of the Bureau of Standards, and the other at the John Fritz Civil Engineering Laboratory at Lehigh University, Bethlehem, Pa. The depth of the beams varied from 18 inches to 10 feet, and the span from 9 feet 6 inches to 20 feet. All the beams in the Pittsburgh tests (series 1) were **I** beams; that is, were **I**-shaped in vertical cross section. Most of those made at Lehigh University (series 4) were also **I**-shaped, but a few beams of rectangular cross section were made. The two beams of series 10 were hollow.

In order to determine with sufficient certainty whether beams of rectangular cross section and long, slender beams would show web stresses and shearing strengths nearly the same as those for the beams tested it would be necessary to test such beams in considerable number. However, this question becomes unimportant practically, because rectangular or slender beams generally will fail by tension or compression in a longitudinal direction, with web stresses so low as to present no unusual problems of web reinforcement.

To resist the heavy concentrations of load the thickness of the webs of the **I** beams was made equal to that of the flanges for short distances near the supports and at the center of the span. For lack of a more exact term these thickened portions of the web are referred to as pilasters. Information on the nominal forms, sizes, and reinforcement of the beams is given in Tables 2 and 3 and Figures 5 to 9, inclusive.

An analysis of web stresses has been developed by Mr. Slater which leads to the same equation between tension in the stirrups and shear in the web as the one which usually forms the basis for the design of web reinforcement. However, the steps in the analysis seem to be more nearly susceptible of rigid demonstration than those ordinarily used in deriving the same formula. The relation between stresses in vertical and those in diagonal reinforcement are also brought out, as well as the relation between the tensile and compressive stresses. It is not supposed that the analysis can hold any more rigidly than the assumptions on which it is based, but it is believed that the experimental verifications of the results of the analysis are as good as may be expected in this type of structure. It is believed that this analysis will form a basis which will assist the reader in classifying the test results, and several references to it are made in the discussion of the data.

Acknowledgment is made to Lehigh University and to members of the faculty, especially Prof. F. P. McKibben and Prof. M. O. Fuller, for the facilities furnished for making and testing the specimens and for hearty cooperation in carrying out the investigation.

II. ANALYSIS

1. GENERAL

Imagine the web of a beam to be replaced by a double system of closely spaced web members represented by *DG* and *GH*, as illustrated in Figure 1, and the longitudinal tension and compression zones to be replaced by the horizontal members *BC* and *AD*, respectively. Imagine all web members to be connected to the longitudinal members at *G*, *H*, etc., or to the stiff posts, *AB* and *CD*, by hinged joints, but to be entirely free from any kind of articulation with each other at intersections. This structure is no longer a beam,

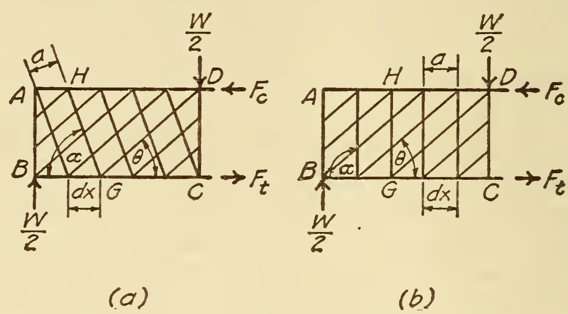

Fig. 1.—*Truss beam with diagonal members*

since by the introduction of the independent elements into the web the direction of the stresses is fixed by the direction of the members and is not determined by the combination of shearing and normal stresses. It is, however, sufficiently like some of the beams which have been tested in this investigation to present an analogy which will be useful in visualizing the action which takes place in these beams.

With the two forces,[1] $\frac{W}{2}$, applied to the half beam as shown in Figure 1, it is obvious that the following condition will exist:

[1] Wherever in this paper the word "force" is used it designates the total force, in the direction indicated, applied to or resisted by the part of the section under consideration. Wherever the word "stress" is used it designates the intensity per unit area of the internal force. For example, the compressive force on any section of a beam under flexure and without direct stress would be the total force on that part of the section of the beam which lies between the neutral axis and the "extreme fiber" in compression. In the same case the term "stress" would designate the intensity per unit area of the compressive force at the point in the cross section which is under consideration. For axial loads the word "force" designates the total force applied to or resisted by the entire cross section of the member. This usage is consistent with the recommendations of Committee E-1 of the American Society for Testing Materials. See Proc. A. S. T. M., 24 (1924), Pt. I, p. 937.

(1) A force F_c causing compression will be applied to AD at D, and a force F_t causing tension will be applied to BC at C.

(2) $F_c = F_t = \frac{M}{jd}$ where M is the applied moment, d is the distance from the compression surface to the center of tension, and j is the ratio to d of the distance between the center of tension and the center of compression.

(3) AD will shorten and BC will lengthen.

(4) The diagonal web members will assist in resisting the shortening of AD and the lengthening of BC.

The rate of change in F_t or F_c along the length of the beam is $\frac{dF}{dx}$. Then $\frac{dF}{dx}jd$ will be the rate of change of moment, $\frac{dM}{dx}$. But

$$\frac{dM}{dx} = V, \text{ the shear} \tag{1}$$

Therefore

$$\frac{dF}{dx}jd = \frac{dM}{dx} = V \tag{2}$$

and

$$dF = \frac{V}{jd}\,dx \tag{3}$$

If dx be assumed to take the finite value s (small enough to permit of considering the total shear V to be constant over the distance s) dF will take a finite value

$$\Delta F = \frac{Vs}{jd} \tag{4}$$

where s is taken as the horizontal spacing of web members and ΔF is the change in total horizontal stress between two web members; that is, in the distance s. The total stresses carried by the members connected at any point, such as G of Figure 1, will then be as shown in Figure 2 and the equations of equilibrium will be

$$F_t + \Delta F - F_{dc}\cos\theta - F_t - F_{dt}\cos\alpha = O \tag{5}$$

and

$$F_{dt}\sin\alpha - F_{dc}\sin\theta = O \tag{6}$$

From (6)

$$F_{dc} = F_{dt}\frac{\sin\alpha}{\sin\theta} \tag{7}$$

From (5) and (6) since

$$\Delta F = \frac{Vs}{jd}$$

$$\frac{Vs}{jd} = F_{dt}\cos\alpha + F_{dc}\cos\theta \tag{8}$$

$$= F_{dt}\left(\cos\alpha + \frac{\sin\alpha}{\tan\theta}\right) \tag{9}$$

It will be seen from equations (5) and (8) that the increment of stress ΔF is equal to the sum of the horizontal components of the diagonal tension and the diagonal compression. If the tension web member is vertical it will have no horizontal component and the diagonal compression must be the greater since ΔF will not be diminished thereby. This illustrates why, as shown by equation (19), page 396, in a beam with vertical stirrups the diagonal compression may be expected to be twice as great as in a beam with stirrups inclined at 45°.

Similarly equation (6) shows that if the tension web member is vertical, as in a beam with vertical stirrups, sin α becomes unity and the total tension in the web member is equal to the vertical component of the force in the diagonal compression member.

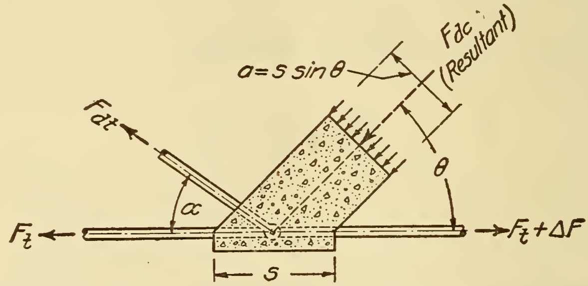

Fig. 2.—*Stresses at junction of web reinforcement with longitudinal reinforcement*

2. SOLUTION FOR DIAGONAL TENSION

If equation (9) may be assumed to apply to a beam in which the concrete takes the diagonal compression and metal web members take the diagonal tension after the concrete has cracked it may be reduced to terms of shearing unit stress, v, and tensile unit stress f_v. Since

$$V = vbjd \text{ and } F_{dt} = A_v f_v$$

$$v = \frac{A_v}{bs} f_v \left(\cos \alpha + \frac{\sin \alpha}{\tan \theta}\right) \tag{10}$$

Letting[2] a be the distance between diagonal tension members at right angles to their direction,

$$s = \frac{a}{\sin \alpha} \text{ and } \frac{A_v}{bs} = \frac{A_v}{ba} \sin \alpha = r \sin \alpha$$

in which r is the ratio of the sectional area, A_v (or volume), of the reinforcement to the sectional area, ba (or volume), of the web which it reinforces. Then

[2] For exactness a should be taken as the distance center to center of adjacent spaces between such members For simplicity it is taken as stated.

$$v = rf_v\left(\cos\alpha\sin\alpha + \frac{\sin^2\alpha}{\tan\theta}\right) \tag{11}$$

$$f_v = \frac{v}{r\left(\cos\alpha + \frac{\sin\alpha}{\tan\theta}\right)\sin\alpha} \tag{12}$$

The test data give some indications as to the propriety of applying this kind of an equation to the relation between shear and tension, and also give indications regarding the value of the angle between the diagonal compression and the horizontal.

It will be seen by reference to Figures 38 and 39 for vertical and inclined web bars, respectively, that the relation between the shearing stress, the tensile stress, and the ratio r appeared to be the same for beams with vertical web bars as for beams with web bars inclined at 45°. This being the case, the same value of v should be obtained from equation (11) when α is 45° as when α is 90°, and from this condition the angle θ of diagonal compression may be determined for these two cases. Substituting $\alpha = 45$ and 90°, successively, in equation (11) and equating the results,

$$v = rf_v\left(\cos 45°\sin 45° + \frac{\sin^2 45°}{\tan\theta}\right) = rf_v\left(\cos 90°\sin 90° + \frac{\sin^2 90°}{\tan\theta}\right) \tag{13}$$

from which

$$0.5 + \frac{0.5}{\tan\theta} = \frac{1}{\tan\theta} \tag{14}$$

and

$$\tan\theta = 1;\ \text{that is, } \theta = 45° \tag{15}$$

The indication, therefore, is that with the diagonal tension members making angles of either 45 or 90° with the horizontal the direction of the resultant compression was approximately 45° with the horizontal. No data were obtained which throw light on the probable direction of the diagonal compression for angles of the diagonal tension members between 45 and 90° with the horizontal, but curves are given in Figure 3 showing the relation between v, rf_v, and α for three quite widely different assumptions as to the direction of the diagonal compression, namely:

(1) That $\theta = 45°$ for all values of α.

(2) That $\theta = 90° - \alpha$; that is, the diagonal compression and the diagonal tension are always at right angles with each other, as in a homogeneous beam

(3) That $\theta = 67\frac{1}{2}° - \frac{\alpha}{2}$; that is, that the maximum diagonal compression lies midway between positions given under (1) and (2)

The indication from that figure is that for angles from 45 to 90° assumption No. 1 is the only reasonable one, and that for angles α less than 45° it makes little practical difference which assumption is used. When θ is made 45° equations (10) and (11) become

$$v=\frac{A_v f_v}{bs}(\cos \alpha+\sin \alpha) \tag{16}$$

$$v=rf_v(\cos \alpha \sin \alpha+\sin^2 \alpha) \tag{17}$$

At the neutral axis of a homogeneous beam the stress in diagonal tension at 45° with the direction of the axis of the beam is equal

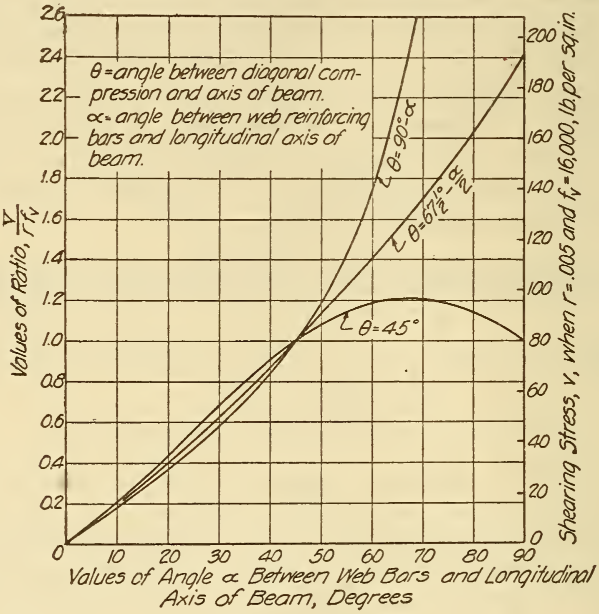

FIG. 3.—*Relation between diagonal tensile stress, shearing stress, and direction of web stresses*

to the shearing stress. If the web thickness is b the total diagonal tensile stress within a short distance a (a being equal to $s \sin 45°$) is vba. In equation (16) the total diagonal tension within the distance s is represented by the term $A_v f_v$. Putting α equal to 45° and s equal to $\frac{a}{\sin 45°}$, it is found from this equation that $A_v f_v$ within the distance a is vba, the same as for a homogeneous beam. While this does not demonstrate the applicability of equations (16) and (17) to a reinforced concrete beam where the web members are placed at a considerable distance apart, it shows that, in a structure similar in many respects to a reinforced concrete beam and susceptible of exact analysis, the diagonal tensile stresses developed are the same as those in a homogeneous beam and gives a feeling of confidence

in the use of this equation for the stress in a concrete beam. The confidence may be the greater because of the fact that the reinforced concrete beam is a structure intermediate, in form, between the homogeneous beam and the trusses assumed for analysis.

In the reinforced concrete beam the concrete will take most of the diagonal tension before the concrete is cracked, and the tests indicate that it may have taken a portion of it even after cracks appeared. In such a beam the concrete must furnish the diagonal compression member.

That the diagonal compression probably took the same direction for $\alpha=45°$ as for $\alpha=90°$ is further indicated by the fact that the direction of the tension cracks was about the same for these cases. See Figures 23 and 52 for vertical stirrups ($\alpha=90°$) and Figures

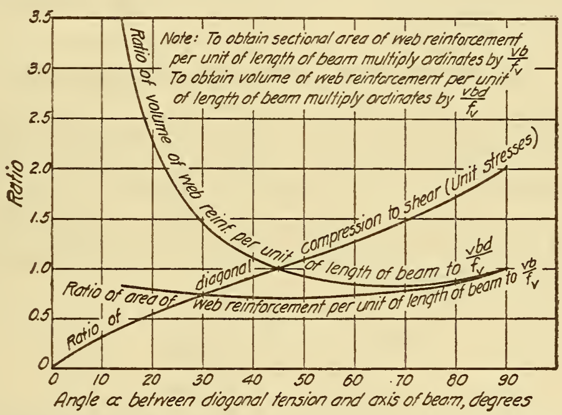

FIG. 4.—*Relation between diagonal compressive stress and shearing stress*

20(*a*) and 53 for inclined stirrups ($\alpha=45°$). This adds justification for extending the anlysis to beams with vertical as well as to those with inclined tension members in the web.

In Figure 4 curves are shown which give proportionate values for the sectional area $\left(\frac{A_v}{s}\right)$ and the volume $\left(\frac{A_v}{s}\,\frac{d}{\sin\alpha}\right)$ per unit length of beam, of tension reinforcement required in the web for various values of the angle, α, of inclination between the web member and the longitudinal axis of the beam. These curves assume the same width, b, of the web, the same depth of beam, d, the same shearing stress, v, at the neutral axis, and the same tensile stress, f_v, in the web reinforcement for all cases. Only the angle, α, is assumed to vary. The values of the sectional area, $\frac{A_v}{s}$, per unit of length were taken directly from equation (16). The volumes were obtained by multiplying the areas, $\frac{A_v}{s}$, by the assumed length, $\frac{d}{\sin\alpha}$, of the web mem-

bers. The curve for areas indicates a comparatively small variation in the sectional area of web reinforcement required for different angles of inclination of the web members. Owing, however, to the variation in length of web member with variation in the inclination, a much greater total quantity of reinforcement is required in the web when the web members are nearly horizontal than when they have an inclination of from 45 to 90°. The tests reported in this paper give data supporting the conclusion that the quantity of reinforcement required when the inclination is 45° is the same as that required to give the same strength when vertical stirrups are used. The increased volumes required for angles less than 45° seem reasonable. The curve indicates that the least volume of web reinforcement would be required when the web members make an angle of about 67° with the axis of the beam. No test data were obtained for angles α other than 45 and 90°.

3. SOLUTION FOR DIAGONAL COMPRESSION

By the same methods as those employed for determining the diagonal tension a solution may be made for the relation between the shear and the diagonal compression. Putting the total diagonal compression, F_{dc}, within the distance s equal to $f_c bs \sin\theta$ in equation (7), where f_c is the compressive stress in the direction of F_{dc}, and solving for f_c it is found that

$$f_c = \frac{v}{\sin\theta\ \cos\theta + \dfrac{\sin^2\theta}{\tan\ \alpha}} \tag{18}$$

If, as is indicated in the previous discussion, θ is always 45°

$$f_c = \frac{2v}{1 + \dfrac{1}{\tan\ \alpha}} \tag{19}$$

According to this equation the diagonal compressive stress is dependent upon the direction of the web reinforcement, but is independent of its amount. Equation (19) and Figure 4 indicate that the diagonal compression is twice as great for beams with vertical stirrups as for beams with stirrups inclined at 45° with the horizontal. Since in beams having no reinforcement for diagonal compression all the diagonal compression is resisted by the concrete of the web, it is reasonable to expect that its intensity should be nearly independent of the amount of tension reinforcement in the web. It is to be noted that all of these indications from the analysis are in conformity with the conclusions based upon study of the test data as pointed out in Sections XIV, page 434, and XVIII, page 446.

III. SCOPE OF TESTS

This investigation included the beams of series 1, series 4, and two beams of series 10 of the tests carried out in the structural laboratory investigations of concrete ships. Series 1 comprised 13 beams with a depth of 4 feet 4 inches and a span of 16 feet, and 1 beam with a depth of 10 feet and a span of 20 feet. Series 4 comprised 156 beams having a span of 9 feet 6 inches. The depth was 36 inches for 147 of these beams and 18 inches for the others. Two hollow beams of series 10 were tested for shearing strength after leakage tests on them had been completed.

Series 1, conducted at the Pittsburgh laboratory of the Bureau of Standards, was very hastily planned and was intended only as an emergency check of the design of web reinforcement as made for the 3,500-ton concrete ship *EF 2*, represented by the *Polias*, built at the Fougner concrete-ship yard in Brooklyn, N. Y., and the *Atlantus*, built at the Government shipyard at Brunswick, Ga. This series contained beams with (1) two-way diagonal reinforcement, (2) vertical web reinforcement only, and (3) vertical and horizontal web reinforcement combined in the form of independent bars not welded and also in the form of welded units.

Series 4, as originally laid out for the concrete-ship investigation had as its main purpose a thorough investigation of (1) the effect of the amount of web reinforcement, and (2) the effect of its direction.

The work covered a period of 16 months, from January, 1918, to, May, 1919. After the signing of the armistice in November, 1918, and especially after January 1, 1919, when the investigation passed to the Bureau of Standards, in order to increase the value of the investigation for ordinary construction and design, series 4 was extended to investigate the effect of the variation in (1) the strength of the concrete, (2) the thickness of the web, (3) the spacing of the stirrups, (4) the method of anchoring the stirrups, (5) strength of the frame of the beam (made up of the upper and lower flanges and the pilasters), and (6) depth of the beam.

The program was not exhaustive, but some information was obtained on the effect of each of these variables.

Table 1 shows groupings of the beams on the bases of type of web reinforcement, web thickness, and richness and average strength of concrete. It shows the total number of beams of each group. The upper half of the table shows the grouping on the basis of web thickness and the lower half on the basis of richness of concrete. The beams shown in the upper half are repeated in the lower half of the table.

Table 2 is a key by means of which information on the form of section for any beam, type, amount, and anchorage of reinforcement, and data of tests may be found easily in Tables 5 to 10 and Figures 5 to 9.

The first column gives in alphabetical order the number by which the beams are identified throughout the report. The initial numeral (1, 4, or 10) in the beam number indicates the series to which the beam belongs. The numeral suffixes 1 to 32 indicate web thickness or mix of concrete, as explained in a note in Table 3. For the letters in the beam numbers no systematic arrangement is possible. The second

TABLE 1.—*Groupings of beams according to type of web reinforcement, web thickness, and richness of concrete*

Range of web thicknesses, inches	No web reinforcement	Type of Web Reinforcement (See FIG. 7): Vertical only (Type A)	Horizontal only (Type B)	Vertical and horizontal (Type C)	Diagonal tension only (Type D)	Diagonal tension and compression (Type E)	Vertical, diag. tension and compression (Type F)	Expanded metal (Types G, H, I)	Total number of beams
0.00	4AG1,2 4AG9,21	4E5,25; 4F5 4G5, 25			4X5				10
1.95 to 2.30		4F4; 4G4			4X4				3
2.40 to 3.45	4J1,2 4J11,21,31	4E1,2,11,12,21,22,31 4F1,2,11,12,21,22,31 4G1,2,11,12,21,22,31 4H1,2,11,12,21,22,31 4BA21,22; 4BB21,22 4BC21,22; 4BD21;22 4YE1; 4YF1; 4YG1; 4YZ1	4S1; 4T1	4A1,2; 4B1,2 4C1,2; 4D1,2 4K1; 4L1 4M1,2; 4N1 4P1; 4R1	4U1,2 4W1,2,21 4X1,2,8,11 4X21,22,31 4X32; 4Y1,2 4Y21; 4AH1,2	1G1 4Z1 4AA1,2 4AB1,2 4AC1,2 4AK2	4CA1,2 4CB1,2 4CC1,2 4CD1,2 4CE1,2	4XA1 4XB1 4XC1 4XD1 4XE1 4XF1 4XG1	106
3.90 to 4.45		1F1; 1I1; 1K1; 1L1 4E3, 4G3		1N1; 1X1 4A3; 4B3; 4C3 4L3; 4R3	4X3	1B1; 1C1; 1E1 1H1; 4AK1			19
4.50 to 5.05	4J3			1M1; 4D3		1A1; 1D1			5
5.65 to 6.05	4J6,26	4E6,7,26; 4G6, 26,27			4X6				9
8.10 to 8.55	4J8,28 4YJ8	4E9,28,29; 4F28; 4G8 4G28,29; 4YE8; 4YF8; 4YG8		10C1; 10D1	4X9				16
11.80 to 12.10		4BE21; 4GA1,21 4GB1							4
Total number of beams	15	73	2	26	23	16	10	7	172

Mix	Series	Water-cement ratio	Strength, lb. per sq. in.	No web reinforcement	(Type A)	(Type B)	(Type C)	(Type D)	(Type E)	(Type F)	(Types G, H, I)	Total No. of beams
1:2*	1		3200		1F1; 1I1; 1K1; 1L1		1M1; 1N1; 1X1*		1A1; 1B1; 1C1; 1D1 1E1; 1G1; 1H1			14
1:2	4	0.55 to 0.62	5400	4J1,2,3,6,8 4AG1,2,9 4YJ8	4E1,2,3,5,6,7,9 4F1,2,4,5 4G1,2,3,4,5,6,8; 4H1,2 4GA1; 4GB1; 4YE1,8 4YF1,8; 4YG1,8 4YZ1	4S1; 4T1	4A1,2,3; 4B1 4B2,3; 4C1 4C2,3; 4D1,2 4D3; 4K1; 4L1 4L3; 4M1,2 4N1; 4P1; 4R1,3	4U1,2,4W1 4W2; 4X1,2,3 4X4,5,6,8,9 4Y1,2; 4AH1 4AH2	4Z1; 4AA1, 4AA2; 4AB1 4AB2; 4AC1 4AC2; 4AK1 4AK2	4CA1,2 4CB1,2 4CC1,2 4CD1,2 4CE1,2	4XA1; 4XB1 4XC1; 4XD1 4XE1; 4XF1 4XG1	103
1:3	4	0.67	4800	4J11	4E11,12; 4F11,12; 4G11 4G12; 4H11,12			4X11				10
1:5	4	0.90	3800	4J21,26,28 4AG21	4E21,22,25,26,28,29 4F21,22,28; 4G21,22,25 4G26,27,28,29; 4H21 4H22; 4BA21,22 4BB21,22; 4BC21,22 4BD21,22; 4BE21; 4GA21			4W21 4X21,22 4YZ1				36
1:9	4	1.05	2100	4J31	4E31; 4F31 4G31; 4H31			4X31,32				7
1:2	10		5050				10C1; 10D1					2
Total number of beams				15	73	2	26	23	16	10	7	172

~ NOTE ~ The volume of aggregate is the sum of the volumes of the fine and coarse aggregates and is not the volume of the mixed aggregate.. Generally the volume of the coarse aggregate was about twice the volume of the fine aggregate.

* All beams of Series 1 were of 1:2 mortar except Beam 1X1 which was of 1:2 gravel concrete.

column in Table 2 gives a reference number by which the beam may be found easily in Tables 7 to 10. For example, to locate beam 4E22 in Table 7 it is found from Table 2 that the reference number is 16. In Table 7 the reference numbers are arranged consecutively, permitting this beam to be located easily.

The third, fourth, fifth, sixth, and seventh columns in Table 2 give letters which refer to sketches in Figures 5 to 9. These sketches show the cross section and type of reinforcement of the beams.

TABLE 2.—*Key to description of test specimens*

[Letters refer to types shown in figs. 5 to 9]

Beam No.	Reference No. tables 5 to 11	Longitudinal section and side elevation	Cross section	Web reinforcement	Flange anchorage	Stirrup anchorage	Beam No.	Reference No. tables 5 to 11	Longitudinal section and side elevation	Cross section	Web reinforcement	Flange anchorage	Stirrup anchorage	Beam No.	Reference No. tables 5 to 11	Longitudinal section and side elevation	Cross section	Web reinforcement	Flange anchorage	Stirrup anchorage
See fig. →		5	6	7	8	9			5	6	7	8	9			5	6	7	8	9
4A1	1	D	H	C	D	A	4H12	53	D	H	A	D	C	4AH2	84	D	H	D	D	A
4A2	1	D	H	C	D	A	4H21	54	D	H	A	D	A	4AK1	86	D	L	E	D	B
4A3	7	D	L	C	D	A	4H22	54	D	H	A	D	C	4AK2	85	D	H	E	D	B
4B1	2	D	H	C	D	A	4H31	55	D	H	A	F	A	4BA21	56	K	F	A	I	B
4B2	2	D	H	C	D	A	4J1	102	D	H	-	D	-	4BA22	56	K	F	A	K	C
4B3	8	D	L	C	D	A	4J2	102	D	H	-	D	-	4BB21	57	K	F	A	J	A
4C1	3	D	H	C	D	A	4J3	106	D	L	-	D	-	4BB22	57	K	F	A	K	C
4C2	3	D	H	C	D	A	4J6	107	F	S	-	D	-	4BC21	58	K	F	A	J	C
4C3	9	D	L	C	D	A	4J8	109	I	J	-	D	-	4BC22	58	K	F	A	K	C
4D1	4	D	H	C	D	A	4J11	103	D	H	-	D	-	4BD21	59	K	F	A	D	C
4D2	4	D	H	C	D	A	4J21	104	D	H	-	D	-	4BD22	60	K	F	A	K	C
4D3	10	D	L	C	D	A	4J26	108	F	S	-	D	-	4BE21	61	G	G	A	K	C
4E1	5	D	H	A	D	A	4J28	111	I	J	-	D	-	4CA1	62	D	H	F	D	A&B
4E2	5	D	H	A	D	A	4J31	105	D	H	-	D	-	4CA2	62	D	H	F	D	A&B
4E3	11	D	L	A	D	A	4K1	51	D	H	C	D	A	4CB1	63	D	H	F	D	A&B
4E5	94	J	K	A	D	B	4L1	31	D	H	C	D	A	4CB2	63	D	H	F	D	A&B
4E6	12	F	S	A	D	B	4L3	36	D	L	C	D	A	4CC1	64	D	H	F	D	A&B
4E7	12	F	S	A	D	E	4M1	20	D	H	C	D	A	4CC2	64	D	H	F	D	A&B
4E9	13	I	J	A	E	C	4M2	20	D	H	C	D	A	4CD1	65	D	H	F	D	A&B
4E11	15	D	H	A	D	A	4N1	30	D	H	C	D	A	4CD2	65	D	H	F	D	A&B
4E12	15	D	H	A	F	A	4P1	32	D	H	C	D	A	4CE1	66	D	H	F	D	A&B
4E21	16	D	H	A	D	B	4R1	33	D	H	C	D	A	4CE2	66	D	H	F	D	A&B
4E22	16	D	H	A	D	A	4R3	37	D	L	C	D	A	4GA1	43	E	R	A	D	G
4E25	95	J	K	A	D	B	4S1	100	D	H	B	D	A	4GA21	48	E	R	A	D	G
4E26	17	F	S	A	D	B	4T1	101	D	H	B	D	A	4GB1	50	E	R	A	D	J
4E28	18	I	J	A	D	B	4U1	67	D	H	D	D	B	4XA1	88	D	H	G	D	K
4E29	18	I	J	A	D	B	4U2	67	D	H	D	D	B	4XB1	89	D	H	G	D	K
4E31	19	D	H	A	D	A	4W1	69	D	H	D	D	B	4XC1	90	D	H	G	D	K
4F1	22	D	H	A	D	A	4W2	69	D	H	D	D	B	4XD1	91	D	H	H	D	K
4F2	22	D	H	A	D	A	4W21	71	D	H	D	D	B	4XE1	92	D	H	I	D	K
4F4	24	D	C	A	D	A	4X1	72	D	H	D	D	B	4XF1	93	D	H	G	D	L
4F5	96	J	K	A	D	A	4X2	72	D	H	D	D	B	4XG1	87	D	H	G	D	K
4F11	26	D	H	A	D	A	4X3	74	D	L	D	D	B	4YE1	6	D	M	A	G	C
4F12	26	D	H	A	D	A	4X4	75	D	C	D	D	B	4YE8	14	I	T	A	G	A
4F21	27	D	H	A	D	A	4X5	99	J	K	D	D	B	4YF1	23	D	I	A	G	C
4F22	27	D	H	A	D	C	4X6	76	F	S	D	D	B	4YF8	25	I	T	A	G	A
4F28	28	I	J	A	F	C	4X8	72	D	H	D	D	B	4YG1	35	D	D	A	G	C
4F31	29	D	H	A	D	C	4X9	77	I	J	D	D	B	4YG8	42	I	T	A	G	A
4G1	34	D	H	A	D	A	4X11	78	D	H	D	D	B	4YJ8	110	I	N	-	H	-
4G2	34	D	H	A	D	A	4X21	79	D	H	D	D	B	4YZ1	21	D&E	H&R	A	D	A
4G3	38	D	L	A	D	A	4X22	79	D	H	D	D	B	1A1	115	B	B	E	B	H
4G4	39	D	C	A	D	A	4X31	80	D	H	D	D	B	1B1	116	B	B	E	C	H
4G5	97	J	K	A	D	B	4X32	80	D	H	D	D	C	1C1	115	H	B	E	B	H
4G6	40	F	S	A	D	E	4Y1	81	D	H	D	D	B	1D1	115	C	B	E	B	H
4G8	41	I	J	A	D	F	4Y2	81	D	H	D	D	B	1E1	116	C	B	E	C	H
4G11	44	D	H	A	D	A	4Y21	83	D	H	D	D	B	1F1	117	B	B	A	B	I
4G12	44	D	H	A	E	A	4Z1	82	D	H	E	D	B	1G1	118	B	B	E	B	H
4G21	45	D	H	A	D	B	4AA1	73	D	H	E	D	B	1H1	116	B	B	E	C	I
4G22	45	D	H	A	E	A	4AA2	73	D	H	E	D	B	1I1	119	B	B	A	C	I
4G25	98	J	K	A	D	A	4AB1	70	D	H	E	D	B	1K1	120	M	B	A	C	H
4G26	46	F	S	A	D	E	4AB2	70	D	H	E	D	B	1L1	120	M	B	A	C	H
4G27	46	F	S	A	D	E	4AC1	68	D	H	E	D	B	1M1	121	B	B	C	C	I
4G28	47	I	J	A	D	F	4AC2	68	D	H	E	D	B	1N1	122	B	B	C	C	H
4G29	47	I	J	A	F	F	4AG1	112	J	K	-	D	-	1X1	123	A	A	C	A	H
4G31	49	D	H	A	D	A	4AG2	112	J	K	-	D	-	10C1	124	L	P	C	L	D
4H1	52	D	H	A	D	A	4AG9	114	J	E	-	D	-	10D1	124	L	P	C	L	D
4H2	52	D	H	A	D	A	4AG21	113	J	K	-	E	-							
4H11	53	D	H	A	E	C	4AH1	84	D	H	D	D	A							

TABLE 3.—*Schematic arrangement of beam numbers for series 4*

[The prefix 4 indicating the series number is omitted from all the beam numbers in this table]

Depth of Beam in Inches	Concrete Mix	Principal Web Bars: Direction, Spacing in Inches	Secondary Web Bars: Direction, Spacing In., Diameter In.	Designed Flange Width in Inches: 17.5	12																												15				17.5				17.5			
			Nominal Web Thickness in Inches	0	0				2		3						3							3			4				12		6				8.5				8.5			
			Diameter of Principal Web Bars in Inches / Exp'nd Metal, lb/ft² Web Surface	0	0	½	⅝	¾	½	⅝	0	⅜	½	⅝	¾	1	1.46	2.56	3.63	5.09	3.63	3.63	3.63	½	⅝	¾	0	½	¾	1	⅜	½	0	½	¾	1	0	½	⅝	¾	0	½	⅝	¾
18	1:5	Vertical, 3.6"	None									BA 21,22					No joints. Sheets hooked over flange rods for anchorage				Vert. lap-joint	Horiz. lap-joint	No anchor hooks								BE21													
		Vertical, 6.5"											BB 21,22																															
		Vertical, 10.0"												BC 21,22																														
		Vertical, 14.4"													BD22																													
		Vertical, 18.0"													BD21																													
	No. of 1¼" Bars in	Upper Flange		12	8				8		8						8							1	1	1	8				8		10				12				0	2	2	2
		Lower Flange		12	8				8		8						8							5	6	7	8				8		10				12				5	12	12	12
36	1:2	Expanded Metal															XC1	XB1	XA1	XG1	XD1	XE1	XF1																					
	1:9	Vertical, 4" c. to c.	None								J31	H31	G31	F31	E31																													
	1:5	Vertical, 4" c. to c.	None		AG21	G25		E25			J21	$H\frac{21}{22}$	$G\frac{21}{22}$	$F\frac{21}{22}$	$E\frac{21}{22}$																	GA21	J26	$G\frac{26}{27}$	E26		J28	$G\frac{28}{29}$	F28	$E\frac{28}{29}$				
	1:3	Vertical, 4" c. to c.	None								J11	$H\frac{11}{12}$	$G\frac{11}{12}$	$F\frac{11}{12}$	$E\frac{11}{12}$																													
	1:2	Vertical, 4" c. to c.	None	AG9	AG½	G5	F5	E5	G4	F4	J½	H½	G½	F½ YZ1	E½									YG1	YF1	YE1	J3	G3	E3		GB1	GA1	J6	G6	$E\frac{6}{7}$		J8	G8		E9	YJ8	YG8	YF8	YE8
	1:9	Diagonal, 4" c. to c.	None										$X\frac{31}{32}$																															
	1:5	Diagonal, 4" c. to c.	None									Y21	$X\frac{21}{22}$	W21																														
	1:3	Diagonal, 4" c. to c.	None										X11																															
	1:2	Diagonal, 4" c. to c.	None			X5			X4			Y½	$X\frac{1,2}{8}$	W½	U½	AH½												X3						X6				X9						
	1:2	Vertical, 5" clear	Diag. 2 Way, 5" clear, ⅜									CE½		CA½																														
		Vertical, 5" clear	Diag. 2 Way, 5" clear, ½										CD½	CB½																														
		Vertical, 5" clear	Diag. 2 Way, 5" clear, ⅝											CC½																														
	1:2	Vertical, 4" c. to c.	Horizontal, 4" c. to c., ⅜									K1	N1		D½														D3															
		Vertical, 4" c. to c.	Horizontal, 4" c. to c., ½								S1		L1		C½													L3	C3															
		Vertical, 4" c. to c.	Horizontal, 4" c. to c., ⅝										P1	M½	B½														B3															
		Vertical, 4" c. to c.	Horizontal, 4" c. to c., ¾								T1		R1		A½													R3	A3															
	1:2	Diag. 4"	Diag. 4"									Z1	AA½	AB½	AC½	AK½														AK1														

NOTE: Beams above heavy line had four 1¼-in. round bars in each upper and lower flange.

NOTE: In Beam BE21 web bars were spaced 8 in. c. to c. ⅞ in. within each vertical face.

NOTE: East half of Beam YZ1 was of I-shaped cross section with 3-in. web. West half was of rectangular cross section with 12-in. web. Reinforcement of web was the same at both ends.

NOTE: In Beams GA1 and GA21 web bars were spaced 4" c. to c. in each of two vertical planes; in GB1 in each of four vertical planes. Planes of outer bars were ¾" within the vertical faces of the web.

NOTE: Except for the numbers of flange bars, YG1, YF1 and YE1 were identical with G½, F½ and E½ respectively, and YJ8, YG8 and YE8 were identical with J8, G8 and E9 respectively.

NOTE: The tension diagonals in Beams CA½ to CE½ were of the same size as the compression diagonals. Verticals are shown as principal web bars.

NOTE: Tension diagonals in Beams Z1, AA½, AB½, AC½ and AK½ same size as compression diagonals.

—GENERAL—

NOTE: Except with Beams AG½, 9, 21, AK1 and X8 numerical suffixes in beam numbers have the following significance:

Suffix	Significance	Mix
1 and 2	web thickness 3 in.	mix 1:2
3	" " 4 in.	" 1:2
4	" " 2 in.	" 1:2
5	no concrete in web	" 1:2
6 and 7	web thickness 6 in.	" 1:2
8 and 9	" " 8½ in.	" 1:2
11 and 12	" " 3 in.	" 1:3
21 to 29	web thickness same as from 1 to 9 respectively.	" 1:5
31 to 32	web thickness 3 in.	" 1:9

All reinforcing bars were round.

NOTE: Beams of Series 1 and Series 10 are not shown in Table 3. NOTE: Actual web thicknesses varied from nominal web thickness and are given in Table 7

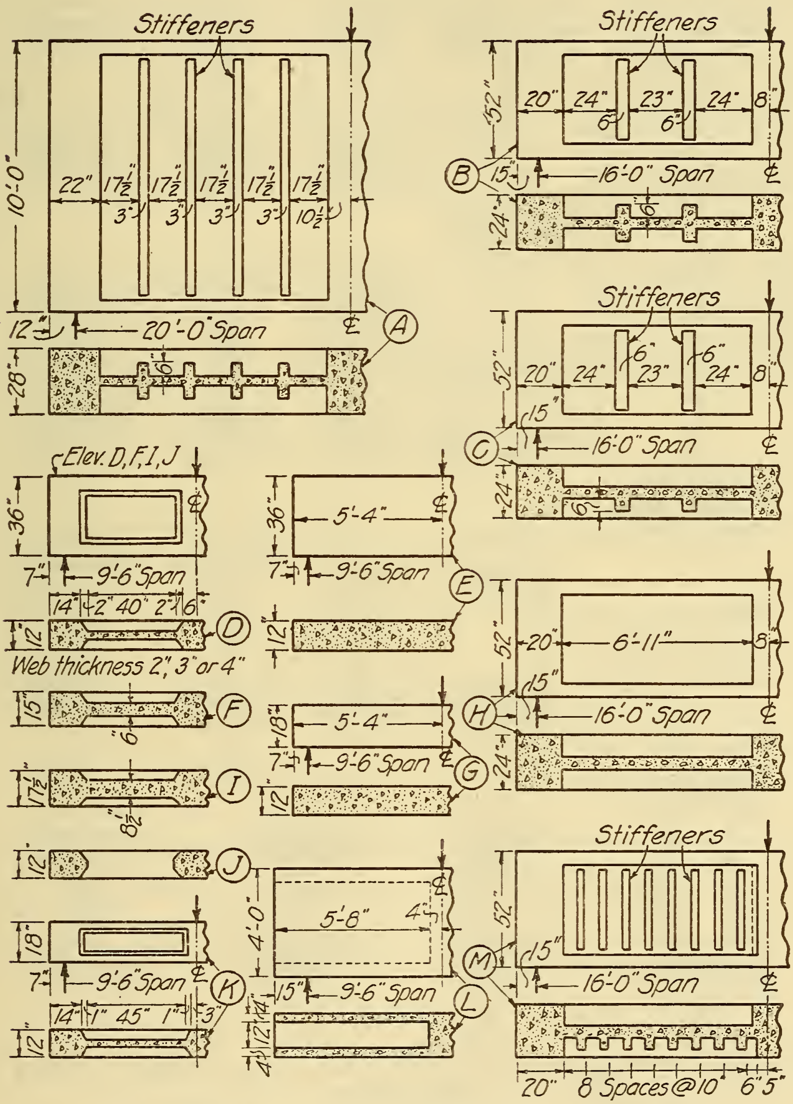

FIG. 5.—*Side elevations and longitudinal sections of test beams*

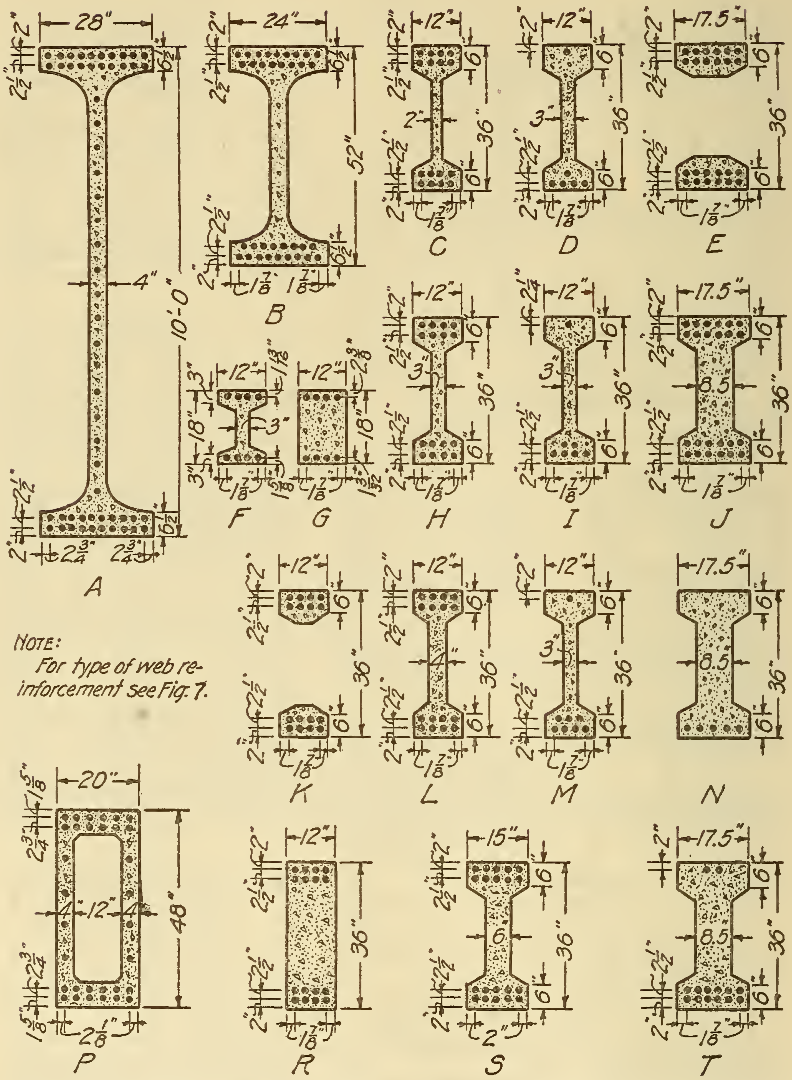

FIG. 6.—*Cross sections of test beams between pilasters showing horizontal reinforcement*

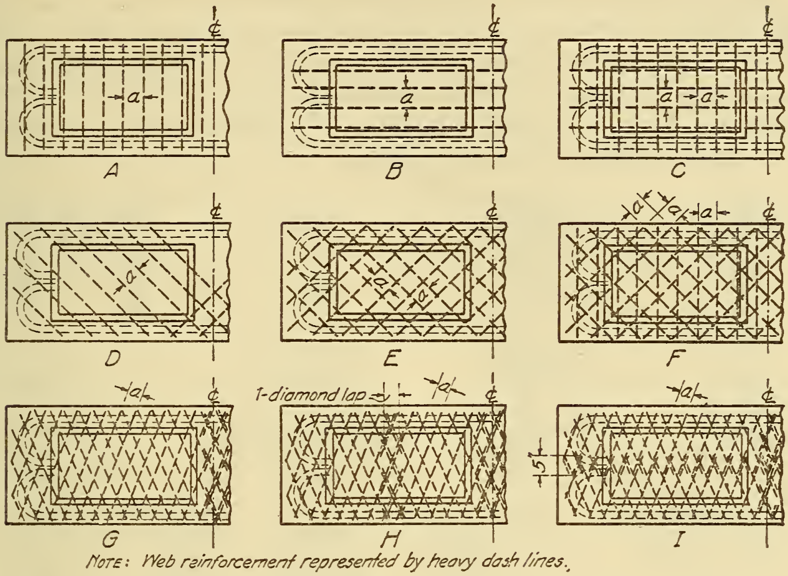

FIG. 7.—*Types of web reinforcement*

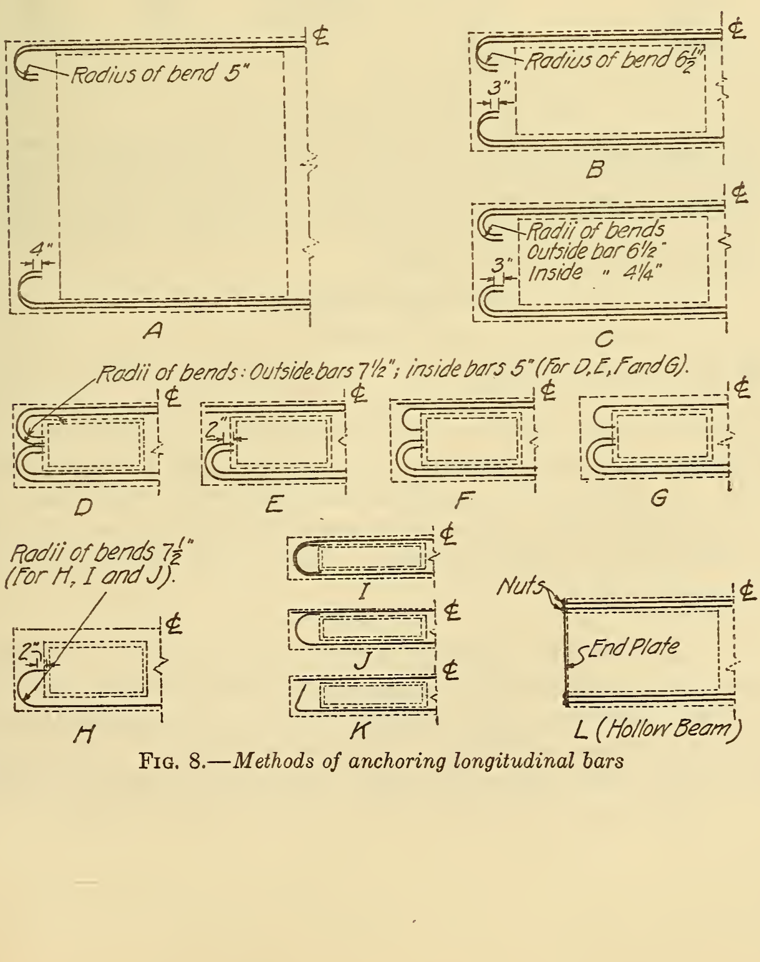

FIG. 8.—*Methods of anchoring longitudinal bars*

Table 3 shows schematically the design data of the beams of series 4, and Tables 7, 8, 9, and 10 give the measured dimensions, the amount of reinforcement, and the principal test results. The dimensions and test results for beams of series 1 and 10 are given in Table 6. Wherever more than one beam of a kind was tested the results given are the averages for beams of the same kind.

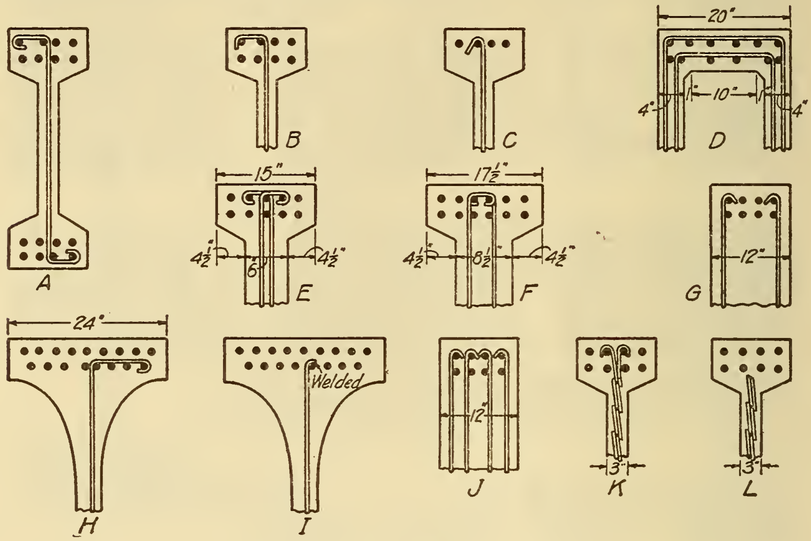

Fig. 9.—*Method of anchoring stirrups*

The letters in Table 2 under column caption "stirrup anchorage" correspond to sketches in Figure 9 and refer primarily to the form of hook used. However, with the exception of *B* and *C* the sketches show also the arrangement of the stirrups. In some cases Table 2 cites sketch *B* or *C* for the form of hook in beams with 6 and 8½ inch webs. In such cases the position of stirrups is given by sketch *E* for 6-inch webs and by sketch *F* for 8½-inch webs. Wherever the web thickness is not given in the sketch the anchorage and arrangement of the stirrups apply to beams of more than one nominal web thickness.

IV. MATERIALS

1. CEMENT

Lehigh Portland cement was used throughout series 1 and 4. Acknowledgment is due the Lehigh Portland Cement Co. for the unusual precautions taken by them with certain shipments to secure uniformity in the cement by spreading out the entire lot of cement on a floor and mixing it thoroughly before bagging it for shipment, and for furnishing to the Bureau of Standards, without cost, the final shipment of 173 barrels of cement. The cement was stored in the laboratory.

2. SAND

The sand used in beams 1A1 to 1L1 (all the beams of series 1 except 1X1), inclusive, was Allegheny River sand taken from storage bins in the winter after dredging had been discontinued and no other material was available. It had been protected from freezing by covering with straw, and some of the broken straw remained

even after the sand had been screened. This sand was mainly siliceous and of good quality, but finer than is desirable, and it contained an admixture of soft coal in objectionable quantities.

For beam 1X1 a much coarser sand from the Ohio River was obtained, which also was mixed with some soft coal particles. These sands were the best available at that time under war conditions.

The first shipment of sand for the beams of series 4, tested at Lehigh University, Bethlehem, Pa., came from the DelawareRiver near Philadelphia. It was fairly coarse, reasonably clean, and composed of round grains, mainly siliceous in character. Other shipments were of washed sand from the Portland, Pa., pits. There is a high clay content in these pits, and even after washing this sand contained from 3 to 6 per cent of clay. It was of about the same fineness as the Ohio River sand used in Pittsburgh, but it was entirely free from coal. Notwithstanding the clay, it was good sand for concrete work and is generally employed for this purpose in Bethlehem. The last shipment of washed sand was from the same source. It was much coarser, and many of the grains were flat and elongated. It had less clay than the previous shipments, but some shale, which was more objectionable.

3. GRAVEL

No gravel was used in any of the beams tested at Pittsburgh (series 1), except in beam 1X1. The gravel used in beam 1X1 was from the Ohio River, and, like the sand used in series 1, it contained considerable soft coal. It was used as received, except that it was screened to remove all pieces over one-half inch in size. This gave an aggregate of the size which it was anticipated would be required for concrete-ship construction.

The first carload of gravel for the beams of series 4 came from the Delaware River near Philadelphia. As received it was dirty, containing both silt and organic matter. Since a colorimetric test gave indication of the presence of humus, the gravel was washed. The resulting aggregate was satisfactory in all respects.

The remaining five carloads of gravel for series 4 came from pits at Portland, Pa., and, like the sand from that source, contained much clay. The clay content ran from 7 to 10 per cent, and this clay clung to the particles very persistently. All the gravel for series 4 was screened, the part passing through a one-eighth inch sieve and also that passing over the one-half inch sieve being rejected. The balance was divided into two lots, one from one-eighth to one-fourth inch in size, termed "fine gravel," and the other from one-fourth to one-half inch in size, termed "coarse gravel."

The screening process reduced the clay content to about 3 per cent in the fine gravel and to about 1 per cent in the coarse gravel. A portion of this gravel was washed, but tests indicated that the effect of this washing on the strength of the concrete was very small.

4. CONCRETE

All of the beams of series 1, made and tested at Pittsburgh, except beam 1X1, were made of 1 : 2 cement mortar, hand mixed. The concrete in beam 1X1 used 1 part cement, 2 parts sand, and 1⅓ parts gravel, machine mixed. Owing to the necessity for speed in carrying out the tests, no precautions were taken to secure refinement of operation in making the concrete. The amount of water necessary to give a concrete which would fill in properly around the reinforcement was judged by eye. The beams were stored in the laboratory where they were made and tested. During a considerable portion of the time low temperature prevailed, even passing below the freezing point at times. On account of these unfavorable conditions, the strength of the concrete varied considerably, The average was about 3,500 lbs./in.^{2}.

In the beams of series 4, made and tested at Lehigh University, Bethlehem, Pa., an accurate record of the weights of all materials entering into the concrete was kept. The aggregate was made up in predetermined proportions of sand, fine gravel, and coarse gravel. The concrete was mixed in a "Wonder" mixer having a capacity of 4 cubic feet, loaned by the Waterloo Cement Machinery Co., of Waterloo, Iowa. Careful placing was necessary to secure filling around the reinforcing bars. This was accomplished by pounding the forms with hammers, but some patching after removal from forms was necessary.

The consistency of the concrete varied widely, as judged by the eye, even when the amount of water, cement, sand, fine gravel and coarse gravel, and the time of mixing were made identical. The consistency was by no means a measure of the strength of the cylinders. The consistency sought was such that when the concrete was dumped from the mixer, falling about 2½ feet, the 3-cubic-foot batch formed a mass about 3 feet in diameter and about 8 to 9 inches high at the center. This consistency was obtained with, perhaps, 60 per cent of the beams. The dimensions of the mass varied, perhaps, from a diameter of 2.5 feet and a depth of 3 inches to a diameter of 5 feet and a depth of 16 inches. The consistencies generally used were well within the requirements for "practical" construction.

The approximate average strengths of the concrete of the various mixes are given in Table 1. These strengths were determined from 8 by 16 inch control cylinders taken during the pouring of the beams.

Within 20 hours after being cast the beams were lifted by crane without damage to the concrete. Twenty-four hours after casting they were covered with burlap and moistened once a day until about eight days before the test date, when they were allowed to dry while the concrete was being cut to expose the steel and the gauge holes were being drilled.

5. REINFORCING STEEL

In series 1 the reinforcing steel came from two sources and was of two grades, the structural grade being used for the flange rods and the hard grade (plain or corrugated) for all web bars. The steel of structural grade showed under tensile test a yield point of about 37,000 lbs./in.^{2}, while the hard-grade bars showed a yield point of about 60,000 lbs./in.^{2} for the one-half inch and 58,000 lbs./in.^{2} for the three-fourths inch size.

The steel for the beams of series 4 was obtained from the Bethlehem Steel Corporation and the Gerber Engineering Co., of Bethlehem. Most of the bars were rolled from the discarded portion of ingots intended for the manufacture of shrapnel. The properties of this steel made it excellent for this investigation. The yield point was somewhat higher than that usually found for hard-grade bars, permitting the beam to carry high loads before failure by tension occurred. Notwithstanding the high yield point, no difficulty was experienced in bending the bars to the required shape. Although purchased under specifications, there was an admixture of bars having a much lower yield point in some of the purchases. Some bars having a low-yield point were inadvertently used in a few of the later beams before its presence was detected, and it is not known just what beams it was used in. The interpretation of the test results for the larger loads is therefore difficult for these beams.

The diamond mesh expanded metal used as web reinforcement for beams 4XA1 to 4XG1 was furnished by the Consolidated Expanded Metal Co., of Braddock, Pa. It was of the type shown in Figure 5 of Technologic Paper No. 233 of the Bureau of Standards. It is probable that its yield-point stress was about 60,000 lbs./in.^{2}, as reported on page 311 of that paper. The yield point was not determined upon the reinforcement used in these beams, but upon the samples furnished by this company at a later date, for the investigation reported in Technologic Paper No. 233.

The yield-point stress of the shrapnel steel bars varied with the size of the bar with some degree of regularity from about 55,000 lbs./in.^{2} for 1¼ inches to about 70,000 lbs./in.^{2} for three-eighths inch bars.

The modulus of elasticity of the steel varied from 30,000,000 to 33,000,000 lbs./in.^{2}. The former value was used in computing the stress from the strains measured in the beams.

The form of stirrups is indicated in Figure 9. The manner of bending the longitudinal bars is indicated in Figure 8. Figures 11 and 12 show the reinforcement for beams of different types. Figure 11 (*a*) shows the reinforcement for a 52-inch beam of 16-foot span. Figures 12 (*d*) and 12 (*e*) show the reinforcement for 18-inch beams of 9 feet 6 inches span. All the other beams shown were 36 inches deep and 9 feet 6 inches in span.

V. TESTING

The beams of series 1 (52 inches in depth) were tested in the 10,000,000-pound hydraulic testing machine at the Pittsburgh laboratory of the Bureau of Standards. The speed of the head of the testing machine is not known, but the load was applied slowly in all cases. On account of the length of time required to make observations of strains, deflections, and openings of cracks a day was generally required in the testing of each beam. The positions of the loads and reactions are shown in Figure 5. The supports of the beams were on pivoted bearings. At one end the bearing was free to roll longitudinally in order to eliminate horizontal thrust at the reactions. Only one load was applied. This was at the center of the span, over a 12-inch length of the upper flange giving a distribution of load sufficient to reduce the probability of failure due to crushing at the point of application of the load.

The 36-inch beams of series 4 were tested in an 800,000-pound Riehle testing machine at Lehigh University. The speed of the head used in the tests was 0.05 inch per minute. Each beam was supported on rocker bearings on a 9-foot 6-inch span and loaded through a spherical bearing block at the center of the span covering an 8-inch length of the upper flange.

The 18-inch beams of series 4, group 4BA, were tested in a 300,000-pound Olsen testing machine at Lehigh University with a speed of testing head of 0.05 inch per minute.

Strains were measured with a strain gauge of the Berry type. This instrument had a ratio between the length of the long arm and the short arm of the pivoted leg of 5 to 1. The gauge length over which strains were measured was 4 inches in most cases. For compression measurements the gauge holes were drilled in brass or steel plugs which had previously been set in the concrete.

In the earlier beams of each type strains were observed on a large number of gauge lines distributed over the web of the beam. It was soon found that the points of maximum stress could be covered by from 11 to 15 gauge lines, and this was the number generally used. Measurements of tension and compression in the flanges and in the web were included. The strain measurements in the flanges were taken at the center of the span in the outside bars of the lower layer of tension reinforcement and in the upper layer of compression reinforcement. The measurements in the web were taken at approximately the one-quarter points of the span and at about mid height of the beam. The deflection was measured to the nearest 0.01 inch at the center of the span at the level of the lower flange, and in a few cases at several points, by means of a fine wire drawn tightly along the side of the beam from support to support and passing over a scale and mirror attached at the center of the beam. (See fig. 72.) By reading on the scale, on a line between the wire and its image in the mirror, parallax was avoided.

An initial load of 5,000 pounds for the 36-inch, 52-inch, and 120-inch beams, and of 2,500 pounds for the 18-inch beams, was applied and the beam was allowed to stand several hours for the plaster bearing to harden before the test was continued. Initial readings with the strain gauge was taken after this load had been in place several hours. The initial deflection was also recorded. Load was then applied in such amounts as to give increments of shearing stress of 100 lbs./in.² and strain and deflection readings were taken at each increment. The first appearance of cracks was noted, and thereafter the widths of cracks were measured at numerous places after each increment of load. For this purpose a celluloid scale graduated to hundredths of an inch and a lens magnifying about 4.5 diameters were used. At a shearing stress of 400 lbs./in.² solid lines following the cracks were painted on one or both faces of the beam. The loading was continued in this manner, with readings at loads corresponding to shearing stresses of 100, 200, 300, 400, 600, 800, 1,100, 1,400, 1,700, 2,000, etc., lbs./in.² until the maximum load was passed, when a final set of readings was taken. Cracks were painted by dashed lines at a shearing stress of 800 lbs./in.² and by dotted lines at the end of the test. The above procedure was developed as the tests progressed, and in a few of the earlier 52-inch beams the size of the load increments and the procedure in marking the cracks varied from that outlined. Interesting and significant phenomena of the tests were carefully recorded, and the crack formation and appearance at failure were both sketched and photographed.

VI. COMPUTATION OF SHEARING STRESSES

Although it is generally recognized that wherever there is a vertical shearing stress there is a horizontal shearing stress of equal intensity, the shearing stress which exists in a beam is probably most often thought of as vertical shear rather than as horizontal shear. This probably is because of the fact that the external forces which set up the shear act in a vertical direction. The conception of the shearing stress as vertical shear may lead many persons to feel that the heavy flanges at the top and bottom of the beam resist a considerable part of the shearing stress set up by the flexure of the beam. However, consideration of the fact that the shearing stress is zero at the top and at the bottom of a homogeneous beam makes it clear that the concentration of material in the flanges could have little effect in resisting the shear. This is more clearly brought out by considering horizontal shear instead of vertical shear. Let Figure 10 represent

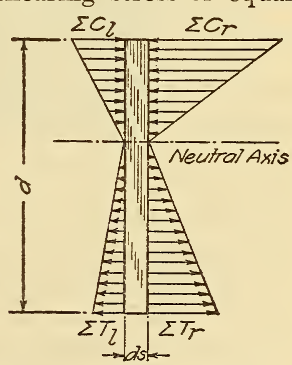

FIG. 10.—*Element of beam showing applied stresses*

a differential length ds in a beam having a depth d, and let the horizontal forces be the flexural tension and compression acting at the sections shown. The sum of the horizontal forces on either face is zero, since all the external forces are parallel to these faces. It is apparent that the total horizontal shear to be resisted at the neutral axis is

$$V_h = \Sigma C_r - \Sigma C_l = \Sigma T_r - \Sigma T_l = \frac{M_r}{jd} - \frac{M_l}{jd} = \frac{M_r - M_l}{jd} \qquad (20)$$

However, since the section is of differential length

$$M_r - M_l = dM = V_v \qquad (21)$$

Therefore

$$V_h = \frac{V_v}{jd} \qquad (22)$$

and

$$v = \frac{V_v}{b' jd} \qquad (23)$$

where

V_h = the total horizontal shearing force at neutral axis,
V_v = total shearing force on a vertical section,
v = shearing stress, either vertical or horizontal,
M_r = moment on section at right of element,
M_l = moment on section at left of element,
b' = thickness of the web at the neutral axis.

When the total load carried by the beam in flexure in the manner indicated is known, it is apparent that no increase in the thickness of the flanges will affect the value of the shearing stress except as it affects the position of the center of gravity of the compressive or tensile forces, and thus affects the value of the moment arm jd.

However, another action comes in, which makes the frame act somewhat as a structure independently of the rest of the beam, and the value for shear given by equation (23) is somewhat in error. This action is best illustrated by considering the beams like 4AG21 (fig. 13), in which there are no webs. The flanges themselves bend and carry the vertical load directly to the end pilasters and down into the supports. With beams which have concrete webs any bending of the flanges independently of the bending of the structure as a whole would introduce some of the same effect. That this effect was present in these beams was shown by the development of vertical cracks on the upper side of the upper flange near the support and on the lower side of it at the edge of the center pilaster. Both of these are positions where, if the structure acted as a unit, compression should be expected. For this case the divergence from proportionality between strain and distance from neutral axis (and therefore

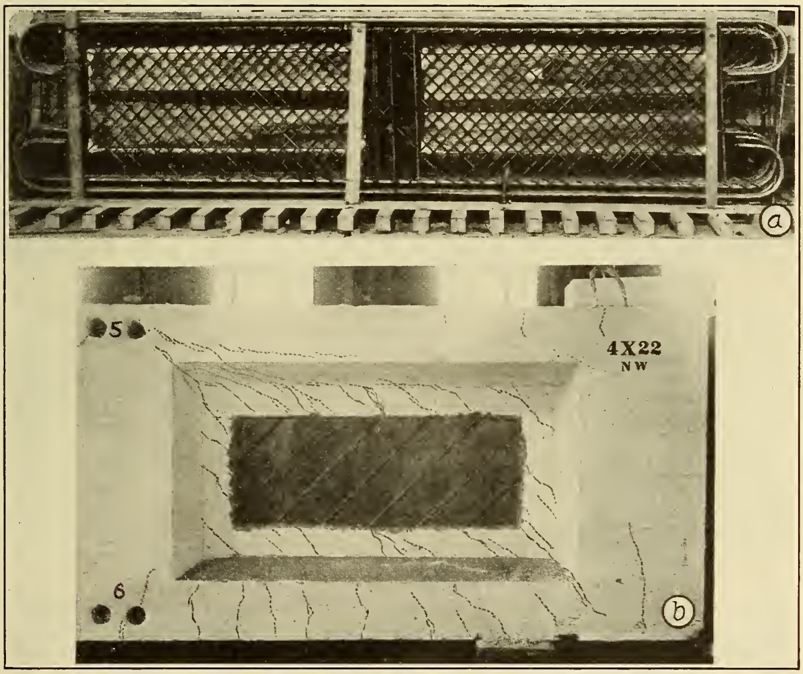

FIG. 11.—*Assembled reinforcement for beams of various types*

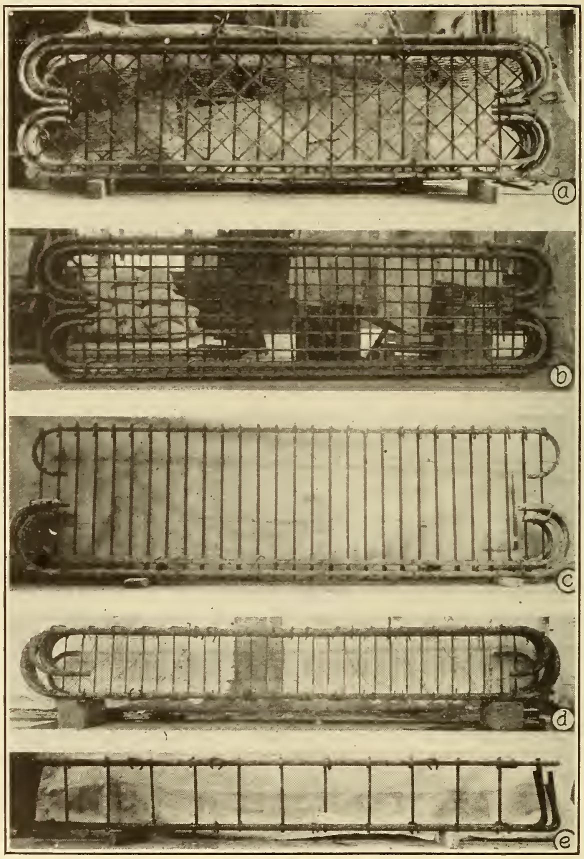

Fig. 12.—*Assembled reinforcement for beams of various types*

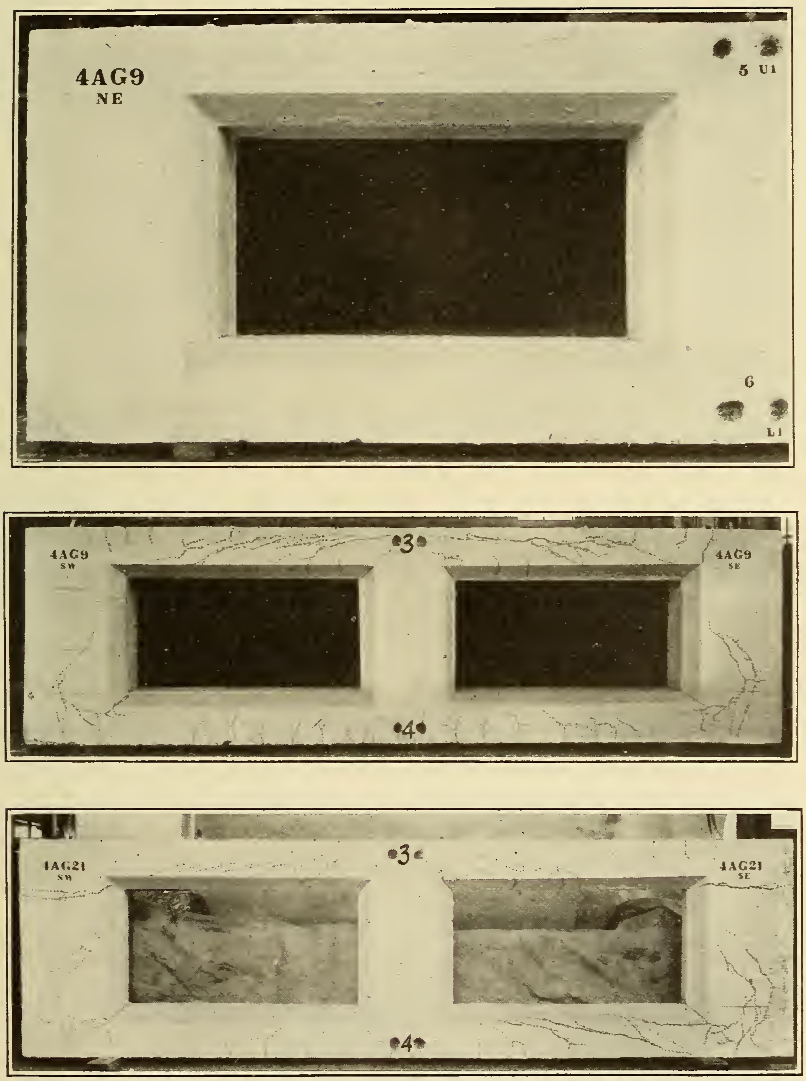

FIG. 13.—*Failure of beams 4AG9 and 4AG21*

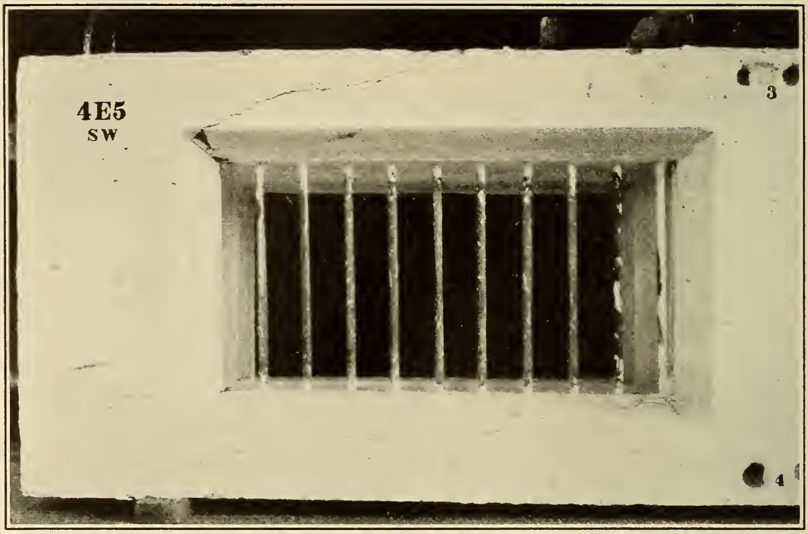

FIG. 14.—*Failure of beam 4E5 with vertical bars, and no concrete, in web*

FIG. 15.—*Failure of beam 4AD1 with inclined bars, and no concrete, in web*

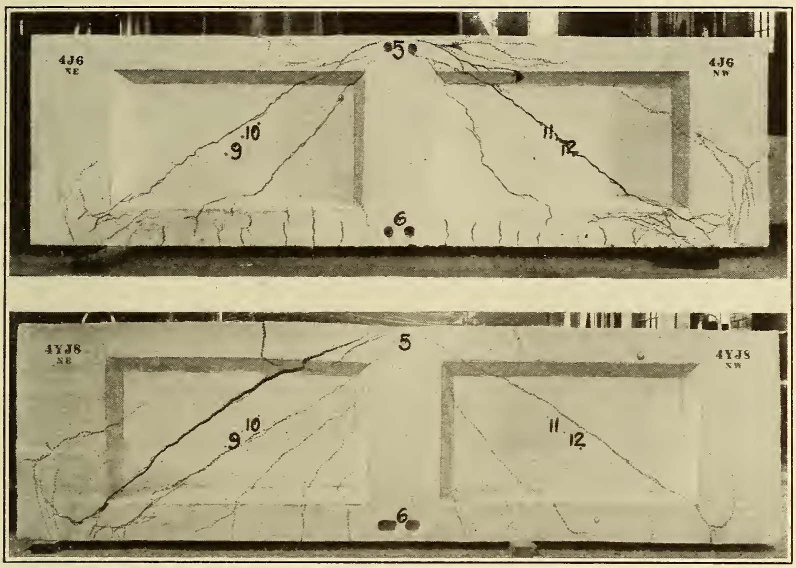

Fig. 18.—*Failure of beams 4J6 and 4YJ8 with 6 and 8.5 inch webs and no web reinforcement*

Fig. 19.—*Failure by horizontal shear of beam 4B2 having vertical and horizontal web reinforcement*

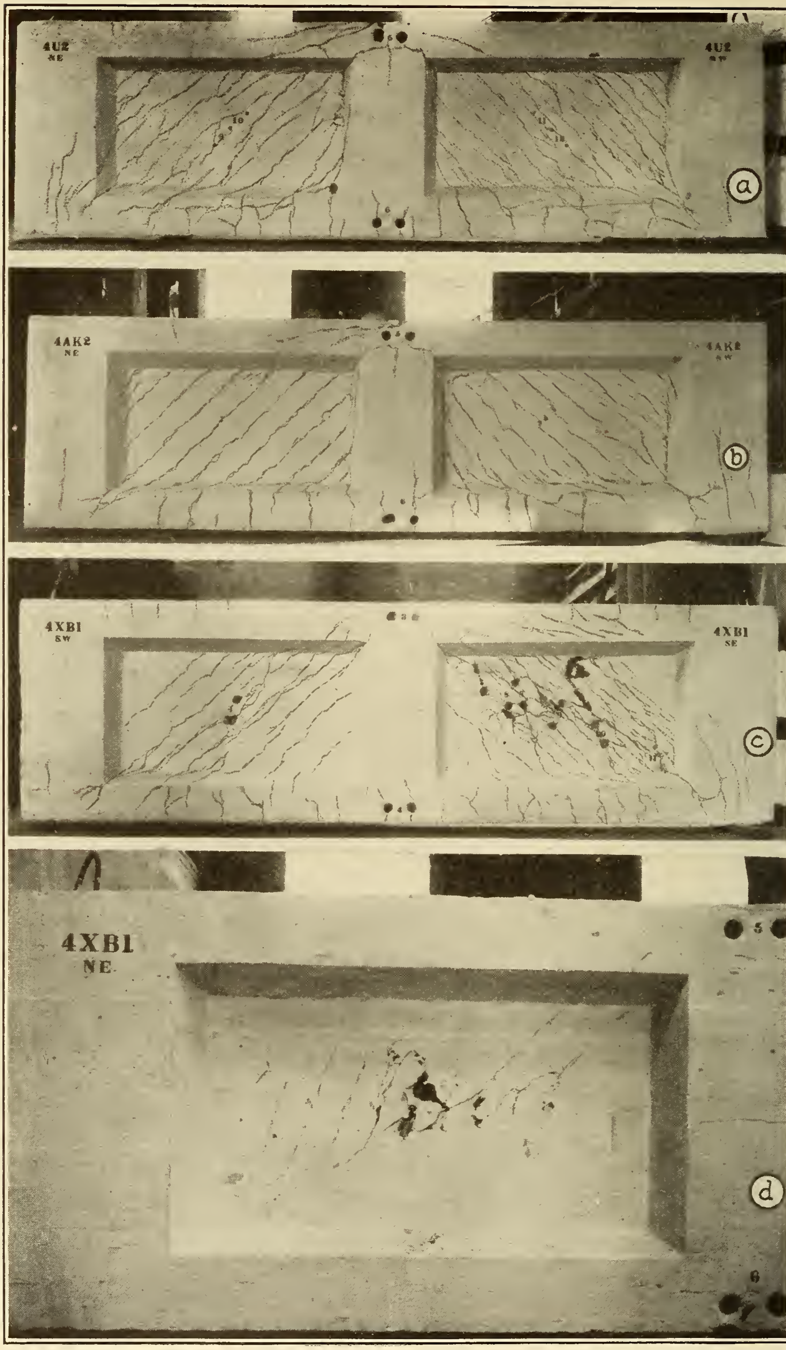

Fig. 20.—*Diagonal tension failures of beams with 3-inch webs*

(*a*) Beam 4U2 with only inclined tension bars in web; (*b*) beam 4AK2 with inclined tension and inclined compression bars in web; (*c*) beam 4XB1 with expanded metal web reinforcement, south side; (*d*) beam 4XB1, north side showing cracks following strands of expanded metal

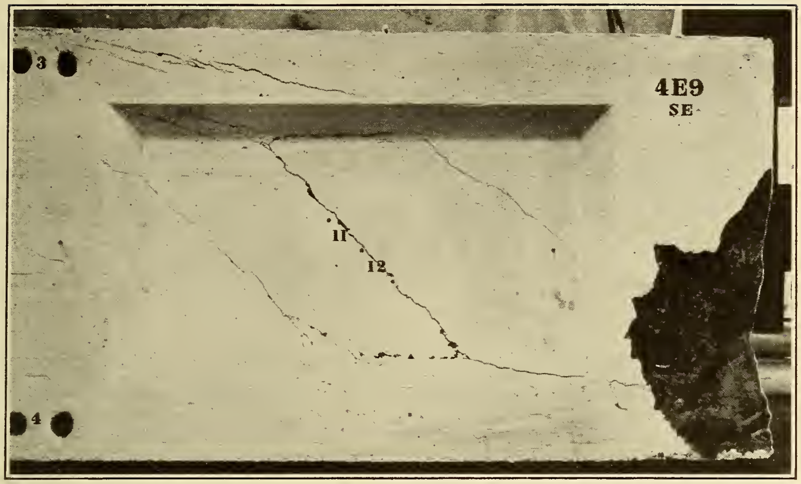

FIG. 21.—*Diagonal tension failure of beam 4E9 having 8.5-inch web and vertical bars*

Corner of beam split off by slipping and straightening of horizontal bars

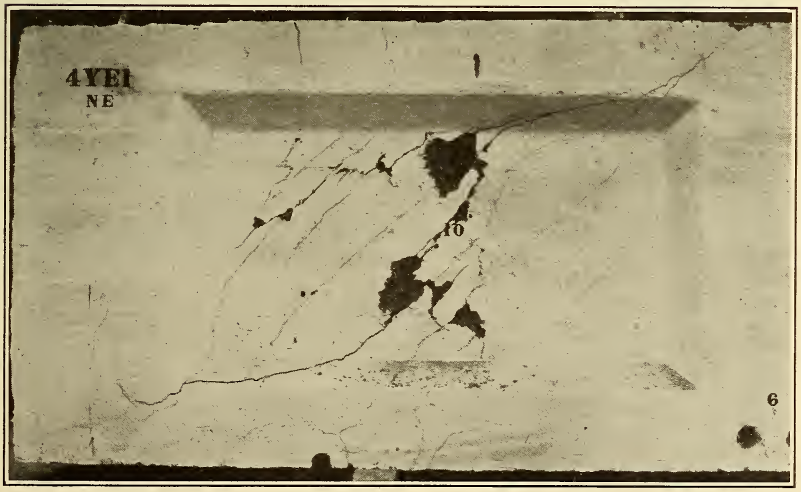

FIG. 22.—*Failure of beam 4YE1 by vertical shear combined with diagonal compression*

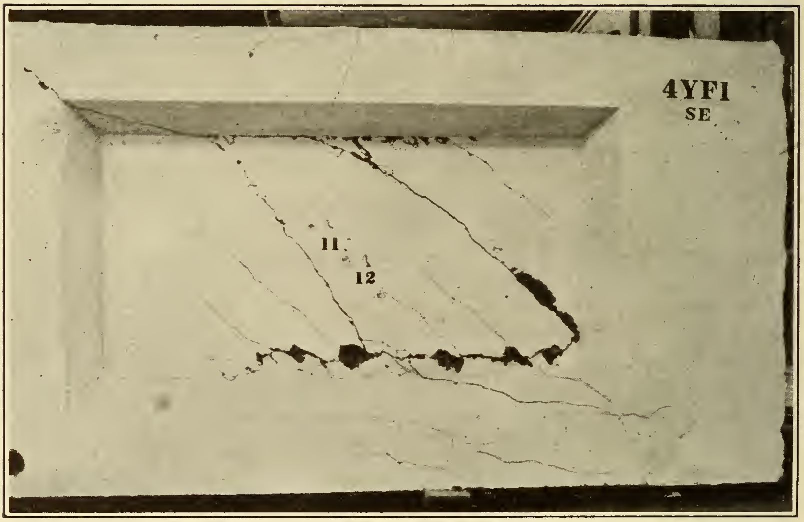

Fig. 23.—*Failure of beam 4YF1 by horizontal shear combined with diagonal tension*

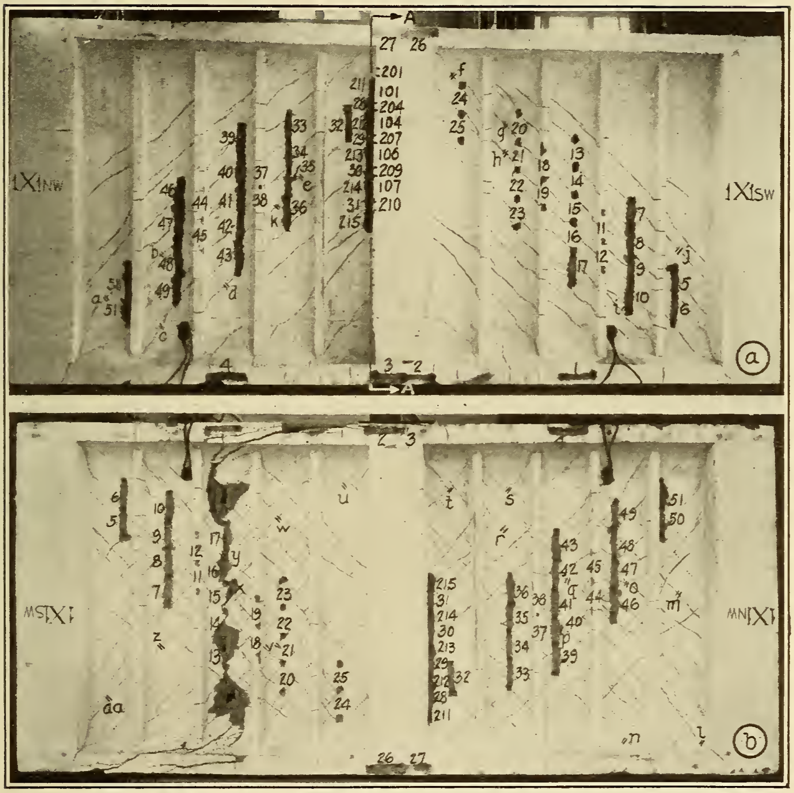

FIG. 24.—*Beam 1X1*

(*a*) West side after 40 applications of load of 640,000 pounds (727 lbs./in.[2] shearing stress) shows crack points and gauge lines used in Figures 26, 29, 30, and 32; (*b*) west side after inversion and loading to failure. Shows crack points used in Figure 26

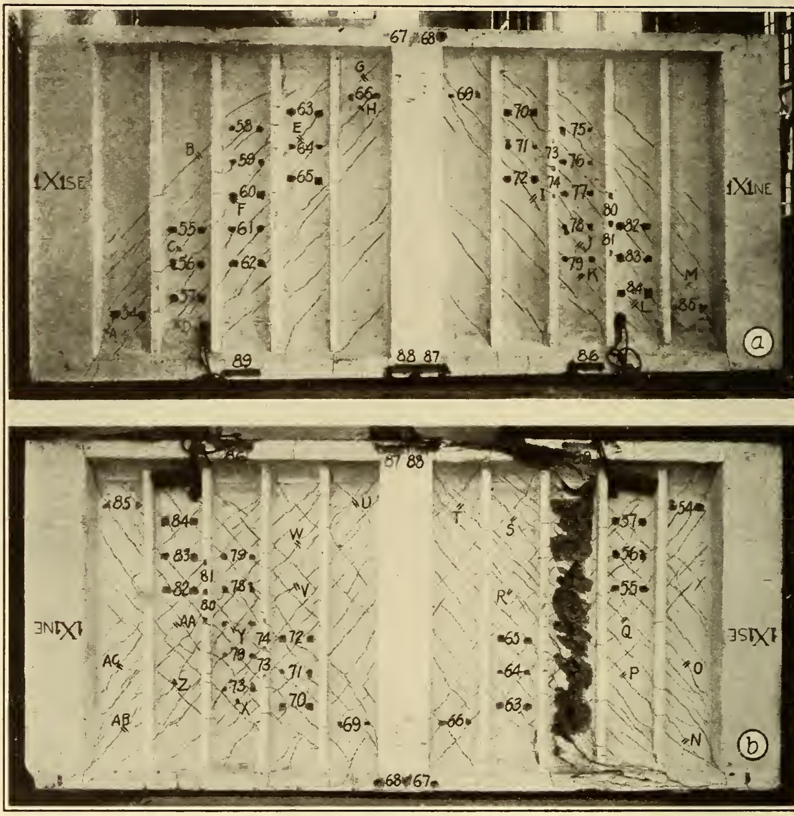

Fig. 25.—*Beam 1X1*

(*a*) East side after 40 applications of load of 640,000 pounds. Shows crack points and gauge lines used in Figures 27, 28, and 31; (*b*) East side after inversion and loading to failure. Shows crack points used in Figure 27.

from the common theory of beam action) must be considerable and the moment which is resisted by flexure on such a section is not known exactly. The values referred to in this paper as "uncorrected" shearing stresses are computed from the total shearing force V by the common formula $v=\frac{V}{b'jd}$. For the study of the test data, however, it is important that a closer approximation to the true shearing stress be reached. The method of making this approximation is given in Section VII, page 412.

TABLE 4.—*Span, depth, and moment arm, jd, for beams of series 1, 4, and 10*

Series	Span	Depth		Ratio of half span to depth d	Number of compression bars in top flange	Web thickness	Flange width	Moment arm, jd
		Over all	To c. g. of longitudinal tension reinforcement					
	Inches	*Inches*	*Inches*			*Inches*	*Inches*	*Inches*
1	180	52	48	2.0	17	3.4 to 5.05	24	46
1	240	120	116.5	1.03	20	3.9	28	113.5
4	114	18	15	3.8	4	3	12	14.95
4	114	18	15	3.8	4	12	12	14.8
4	114	36	32	1.78	1	3	12	28.9
4	114	36	32	1.78	8	0	12	29.5
4	114	36	32	1.78	8	2	12	29.5
4	114	36	32	1.78	8	3	12	29.5
4	114	36	32	1.78	8	4	12	29.5
4	114	36	32	1.78	8	12	12	28.7
4	114	36	32	1.78	10	6	15	29.5
4	114	36	32	1.78	10	8.5	17.5	30.0
4	114	36	32	1.78	2	8.5	17.5	28.4
4	114	36	32	1.78	12	8.5	17.5	29.6
10	114	48	45	1.27	12	8.2	20	42.3

The values of jd calculated for the beams of series 1 and 4 are shown in Table 4. In computing these values the center of tension was assumed to be at the center of gravity of the cross-sectional area of the reinforcement. The actual shape of the section of the beam was used in determining the position of the center of gravity of the longitudinal compressive stresses, the cross-sectional area of the compression reinforcement was considered to be 11 times as effective as an equal area of concrete, and the longitudinal compressive strain [3] was assumed to be proportional to distance from the neutral axis. It will be seen that the variation in jd for the 36-inch beams was slight, and the value of 29.5 inches was used for calculation of the shearing stresses in all beams of 36-inch depth. For the 18-inch beams the value of jd used was 15 inches. For the 52-inch beams (series 1) jd was taken as 46 inches, and for the 10-foot beam 1X1 jd was taken as 113.5 inches.

[3] Wherever in this paper the word "strain" is used it denotes the change of length per unit of length. See report of Committee E-1, Proceedings American Society for Testing Materials, 24 (1924), Pt. I, p. 937.

In all tables the values of shearing stress on the "gross sections" are obtained by dividing the total shearing force by the product of the web thickness and moment arm *jd*. In obtaining the value given in Table 8 under "Net vertical section" the web thickness used was the total web thickness minus the diameter of the vertical stirrups. Correspondingly, for the net horizontal section the web thickness used in the computation was the total web thickness minus the diameter of the horizontal web reinforcing rods.

VII. CORRECTIONS FOR STIFFNESS OF FRAMES

It is apparent that the flanges can not carry load as beams independently of the rest of the structure without developing large deflections, and for a given deflection it seems reasonable to assume that for a beam having a flange width, b, and a web thickness, b', the additional load due to the stiffness of the frames was equal to the load carried by a beam having no web and having a flange width equal to $b-b'$. On this basis a determination has been made of corrections for the shearing stresses.

Generally, however, beams without webs were not available in which the total flange width was equal to the added flange widths for the beams having concrete webs. Consequently it has been necessary to assume that the load carried by a beam without a web would be proportional to the width of the flanges. This should be approximately true, since the number of bars in the flanges was proportional to the width of the flange. A comparison of beams 4AG1 and 2 (having 12-inch flanges) with 4AG9 (having 17.5-inch flanges) shows that the assumption of this relation is approximately correct. The corrections determined independently from these beams are found to agree closely. The method of determining the corrections is here given in detail.

For any given deflection,

$$\frac{W'}{W_{AG}}=\frac{b-b'}{b_{AG}} \tag{24}$$

from which

$$W'=\frac{(b-b')\,W_{AG}}{b_{AG}} \tag{25}$$

where

W_{AG} = load carried by beam with no concrete in web (beams 4AG1, 2, 9, and 21),

W' = load carried by frame action of flanges and pilasters of a beam with concrete in the web,

b = flange width of beams with concrete in web,

b' = web thickness of beam,

b_{AG} = flange width of beam with no concrete in web.

The load W as defined above might be used as a correction to be applied to the load carried by any beam with concrete in the web, but it is more convenient in application to state this correction as a shearing stress instead of a total load.

The shearing stress v', which should be used as an adjustment to correct the shearing stress calculated for any beam having the web thickness b' and the flange thickness b, is

$$v' = \frac{W'}{2b'jd} = \frac{b-b'}{b_{AG}} \frac{W_{AG}}{2b'jd} \tag{26}$$

In Figure 16 the deflections for the beams without concrete in the web (beams 4AG1, 2, 9, and 21) are plotted as abscissas and the

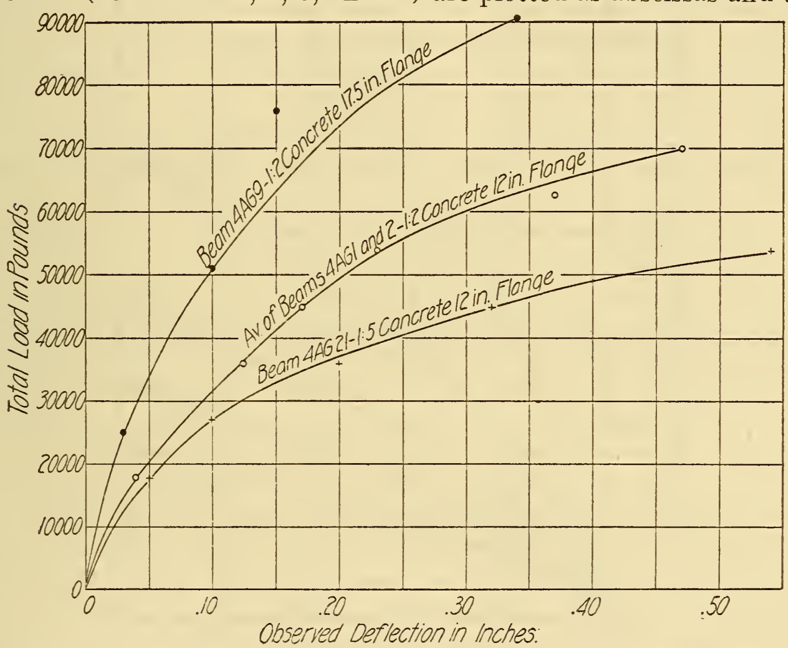

Fig. 16.—*Load-deflection curves for beams without concrete in the web*

total loads as ordinates. Figure 17 shows shearing stress corrections arrived at in this way for beams of the various web thicknesses used in these tests.

Before arriving at the above method of correcting for the frames several attempts were made to determine on a more exact analytical basis what correction should be made for the additional strength due to the flanges. Beams 4AG1, 2, 9, and 21 were made for the purpose of assisting in this analysis. In addition, with the hope of obtaining a measure of the horizontal shear at various heights in the web and flanges, strain readings were taken at more than 150 places on one beam having a web, but the sensitiveness of the in-

struments apparently was not sufficient to determine the differences in stress at different depths. The development of diagonal tension cracks high in the web also made trouble in this study. Analysis of the deflection measurements taken on several beams gave a fair idea of the shape of the elastic curve for the flanges of the beams with concrete in the web and for those without (see figs. 72 and 73

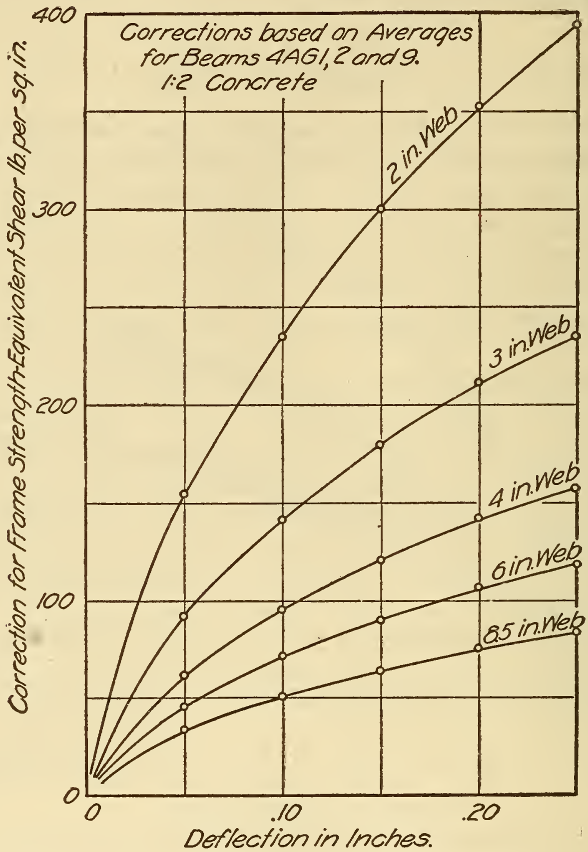

FIG. 17.—*Shear corrections for various web thicknesses*

The corrections shown in this figure were subtracted from the shearing stresses computed by formula 23 and the corrected shearing stresses are given in Tables 6, 8, and 10 and in Figures 38, 39, 44, and 45 (*a*). The corrected shearing stresses are believed to represent approximately the forces per square inch of web section transferred from the tension flange to the compression flange

and Sec. XXIV, p. 467) and gave indications regarding the amount of the correction, but the analysis was not conclusive because of doubt as to the correctness of certain assumptions which are involved. Other studies of the data were made, but none seemed to furnish a more reliable method of determining the correction for the frames than the method stated in the preceding paragraphs.

VIII. PHENOMENA OF TESTS

In the course of a test the first effect on the beam noticeable to the eye was generally the formation of cracks in the web. In the beams without stiffeners the cracks usually were inclined at about 45° and started near the center of the web, lengthening rapidly until they extended to the top and bottom flanges. In some of the 52-inch beams of series 1 which had stiffeners 29 inches center to center the first cracks to appear took the direction, approximately, of the diagonal of the panel formed by adjacent stiffeners and the upper and lower flanges. Cracks which formed later, however, generally took a direction of about 45° with the horizontal and passed through the stiffeners which were within their range. This is illustrated in beam 1F1 (fig. 53). The large diagonal crack, *A*, which passes diagonally across the middle panel of the right half of the beam, was the first crack to appear in that beam. The other cracks, which make an angle of about 45° with the horizontal and which pass through the stiffeners, appeared later in the test. Generally the 45° cracks which appeared later were the ones to open widest in the failure of the beam. In beam 1F1 the steeper crack, *A*, opened widest in the failure of the beam. It seems that the presence of the stiffener affected the direction of the stress, and therefore that of the crack. In some instances the first crack to appear was nearly vertical, and in this case initial stresses due to shrinkage of the concrete on hardening probably determined the direction of the first crack.

After the formation of the first diagonal crack the order of appearance of other cracks varied somewhat in different beams.

Vertical cracks in the top of the top flange near the edge of the end pilaster generally appeared considerably before the maximum load was reached. This indicates that there was a secondary action of the flanges and pilasters as of a frame, which would be expected to relieve the web of carrying all the load as a shearing stress. The amount of this relief is considered in Section VII, page 412, under the heading "Corrections for stiffness of frames." Diagonal cracks in the flanges also appeared at high loads.

The formation at high loads of cracks following the contour of the hooks of the lower longitudinal bars indicated a tendency of those bars to straighten under the tensile stresses developed in them. Such cracks are well shown at the left end of beam 4YJ8 (fig. 18), and they are discussed more fully in Section XXII, page 457.

At the maximum load, or slightly earlier, longitudinal cracks were frequently found on the top of the top flange extending from the edge of the loading block toward the end of the beam. Generally these cracks were from 8 to 12 inches long, but sometimes they were as much

as 30 inches long. In some instances cracks crossed the top flange diagonally, indicating a twisting action of the beam. No photographs are available which show these cracks.

IX. CHARACTERISTICS OF TYPES OF FAILURES

1. GENERAL

In Table 7, under the caption "Manner of failure," symbols are given which represent the opinion of the test observer as to the nature of the failures. In many cases indications of more than one type of failure were present, and some judgment was exercised in deciding the primary cause of failure. In still other cases the appearance of the failure may not truly indicate its character. The relation between shear and diagonal compression is so intimate that a pure shear failure in the concrete of the web of a beam would not be possible. Bearing this in mind, it will be recognized that in all failures indicated as due to either vertical or horizontal shear the important consideration may have been a diagonal compression.

2. VERTICAL SHEAR (V. S.)

See Figure 24. Failure occurred by crushing and spalling in a vertical plane or zone in the face of the web directly opposite a vertical web bar. This may be the result of a large number of adjacent local diagonal compression failures in the portion of the web where the thickness of concrete was reduced by the presence of a rod. In the case of beam 1X1 the failure involved the complete collapse of a vertical strip of the web between two adjacent vertical web bars.

3. HORIZONTAL SHEAR (H. S.)

Horizontal shear failure is illustrated by Figure 19, which shows beam 4B2 after failure. Failure occurred by crushing and spalling along a horizontal plane at the level of horizontal web reinforcing bar. The crushing was apparently due to the reduction, by the presence of the horizontal bar, of the section of the concrete web which resisted the horizontal shearing stresses.

4. DIAGONAL TENSION (D. T.)

Failure was caused by overstrain of the web reinforcement or by diagonal cracks in the web if no web reinforcement was present. (See figs. 18 and 20.) In general, the observed strain in the web reinforcement was at or close to the yield point at the readings next before the failure. Generally one or two of the diagonal tension cracks in the web increased greatly in size and passed through the flanges. The diagonal tension failure in the flanges was secondary and generally occurred after the load on the beam had declined considerably below the maximum.

5. LONGITUDINAL TENSION (L. T.)

Failure was due to exceeding the yield-point stress in the longitudinal reinforcement at the bottom of the beam in the center.

6. DIAGONAL COMPRESSION (D. C.)

See Figure 22. Failure was due to the crushing of the web concrete in a plane nearly normal to the direction of the diagonal tension cracks. These failures generally started at the holes cut in the web to permit the taking of extensometer readings on the web reinforcement.

7. COMBINED VERTICAL SHEAR AND DIAGONAL TENSION (V. T.)

This type of failure occurred only in the beams having web bars extending in three directions—vertically, at 45° for tension, and at 45° for compression. In this failure the stress in the inclined tension bars of the web passed the yield point considerably before the stress in the vertical web bars reached the yield point. The concrete crushed in much the same manner as is described under "Vertical shear" failure.

8. SLIPPING OF WEB REINFORCEMENT (SLIP)

The failure of beam 4AK2, shown in Figure 20 (*b*), is believed to have been due to slipping of the web reinforcement. In two beams having three-fourths inch diagonal stirrups and in all four beams having 1-inch diagonal stirrups there was evidence that led the observer to report a slipping of the web bars at the point where they entered the upper flange. The outward evidence was a crushing on a horizontal line at the juncture of the web and the center pilaster and near the top of the pilaster. In general, high tensions were found in the web reinforcement in this corner of the web. The point of highest stress on any stirrup was much nearer the top of the beams for the stirrups close to the central pilaster than for those at the quarter point of the beam, where the maximum stirrup stresses were generally found.

In beams with three-fourths inch vertical stirrups and 3-inch webs the failure is noted as diagonal tension in some instances, although the stress in the web reinforcement was less than the yield-point stress. It is possible that these failures also may have been due to slipping of the web bars, although they were not so reported at the time of test. In beams having 6-inch or 8.5-inch webs the three-fourths inch vertical stirrups were stressed to the yield point in all cases and true diagonal tension failures occurred.

9. CRUSHING OF PILASTER (C. P.)

In two hollow beams designed for leakage tests and later tested in shear the center pilasters were not of sufficient section to stand the

71966°—26†——3

direct compression. The final failures occurred by diagonal crushing in the pilaster. No figure is available which illustrates this failure.

X. TESTS OF BEAM 1X1 (10 feet deep)

1. GENERAL

The design and test of beam 1X1 were both so different from those of the other beams reported here that the beam and its test are described and discussed in considerable detail.

Beam 1X1 was 20 feet long and 10 feet deep. The web was 3.9 inches thick. The web reinforcement consisted of three-fourths inch round bars placed vertically 4½ inches center to center and three-fourths inch round bars placed horizontally 6 inches center to center. The vertical and horizontal bars were electrically welded at all intersections. The concrete consisted of 1 part cement, ⅔ parts sand, and 1⅓ parts gravel. All of the gravel passed a one-half inch screen. The ratio by volume of the water to the cement was about 0.62. The strength of the 6 by 12 inch concrete control cylinders was 4,620 lbs./in.^{2} at the time of the test of the beam. The modulus of elasticity determined from the control cylinders was 4,840,000 lbs./in.^{2}

The purpose in this test was to verify on a larger scale than was used in the other beams of series 1 the shearing resistance of the shells of concrete ships. The vertical ribs (stiffeners) correspond to the closely spaced frames of the concrete ships. They were much smaller than the ship frames, but were intended to be large enough to resist buckling under compressive stresses. They were 3 inches thick and projected 6 inches from the surface of the web on opposite sides of the beam.

Beam 1X1 was made at the Pittsburgh branch of the Bureau of Standards on April 8, 1918. The test began on May 27 in the 10,000,000-pound testing machine and was completed on May 31, 1918. The beam was supported at the ends and was loaded by means of a single concentrated load at the center. The usual provision of roller bearings at one support and at the center load was employed in order to avoid arching of the load to the supporting girder.

A load of 640,000 pounds (giving a computed maximum shearing stress of 727 lbs./in.^{2}) was applied, removed, and reapplied 40 times in order to ascertain whether a marked increase of crack widths and stresses in the web reinforcement would occur under such conditions. After 40 applications of the load of 640,000 pounds the beam was inverted in the testing machine and load was applied in the inverted position to determine whether the previous test had affected the ability of the beam to resist stresses when it was loaded in the reverse direction. Strain-gauge readings of deformations were

taken on about 105 gauge lines. These gauge lines were distributed over the web reinforcement and longitudinal reinforcement as shown in Figure 24 (*a*) and (*b*), and over the central pilaster, as shown in Figure 32. The latter gauge lines were used to investigate the load distribution in the central pilaster.

2. SCHEDULE OF TESTS

The testing schedule was as follows:

May 27, 7.30 a. m. Zero readings taken.

May 27, 10.45 a. m. Load of 160,000 pounds applied.

May 27, 11.19 a. m. Load of 320,000 pounds applied.

May 27, 1.37 p. m. Load of 480,000 pounds applied.

May 27, 2.32 p. m. Load of 640,000 pounds applied.

May 27, 3.58 p. m. Load released to 10,000 pounds for night.

May 28, 8.53 a. m. Load of 640,000 pounds applied five additional times, with release to 200,000 pounds between applications. Six applications in all. Readings taken.

NOTE.—After each application of 640,000 pounds the load was released to 200,000 pounds, except as noted.

May 28, 9.33 a. m. Sixth to eleventh application of load of 640,000 pounds.

May 28, 10.12 a. m. Twelfth to fifteenth application of load of 640,000 pounds.

May 28, 10.41 a. m. Load released to 10,000 pounds and readings taken.

May 28, 11.10 a. m. Sixteenth to twentieth application of load of 640,000 pounds.

May 28, 11.54 a. m. Twenty-first to twenty-fifth application of load of 640,000 pounds.

May 28, 1.15 p. m. Twenty-sixth to thirtieth application of load of 640,000 pounds.

May 28, 1.44 p. m. Thirty-first to thirty-fifth application of load of 640,000 pounds.

May 28, 2.12 p. m. Thirty-sixth to fortieth application of load of 640,000. pounds.

May 28, 3.15 p. m. Load released to 10,000 pounds and readings taken.

May 28, 4.30 p. m. Load entirely released preparatory to removing beam from machine.

May 29. Beam taken from machine and replaced in inverted position.

May 29, 2.35 p. m. Load of 10,000 pounds applied and new zero readings taken.

May 31, 8.16 a. m. Load of 160,000 pounds applied.

May 31, 8.45 a. m. Load of 320,000 pounds applied.

May 31, 9.38 a. m. Load of 480,000 pounds applied.

May 31, 10.16 a. m. Load of 640,000 pounds applied.

May 31, 11.36 a. m. Load of 800,000 pounds applied.

May 31, 1.05 p. m. Load of 960,000 pounds applied.

May 31, 1.37 p. m. Load of 1,120,000 pounds applied.

May 31, 2.08 p. m. Load of 1,208,000 pounds applied.

May 31, 2.30 p. m. Beam failed at load of 1,363,000 pounds, which dropped off to 220,000 pounds after failure.

3. EXPLANATION OF FIGURES 26 TO 31

The crack widths measured at the points indicated by letters in Figures 24 and 25 on both sides of beam 1X1 are shown in Figures 26 and 27. Each graph is given a letter corresponding to the letter by which the crack is marked in the photograph. The same letter is placed on the vertical line which passes through the origin of the graph for the crack under consideration. In both figures there are three series of graphs. The lowest graph shows the progressive increase in width of crack as the load was increased progressively up to 640,000 pounds. The next higher graph shows the increase (or decrease in some cases) of crack width during the progress of 40 repetitions

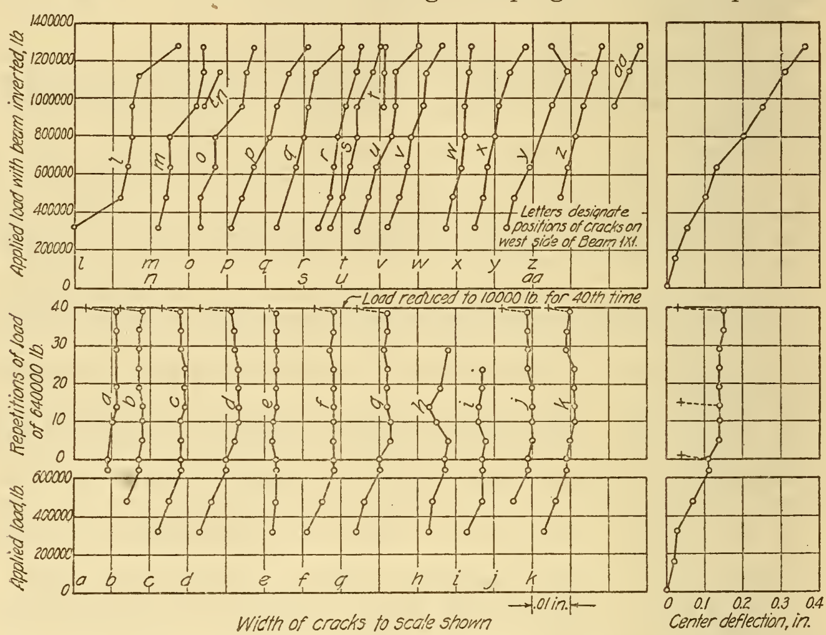

Fig. 26.—*Crack widths measured on west side of beam 1X1 at points marked by letters in Figure 24 (a) and (b); zero of crack width at line marked with same letter as crack*

(41 applications) of the 640,000-pound load and one repetition of the 10,000-pound load. The third (upper) series of graphs are for the cracks which occurred on the same side of the beam after the beam was inverted. For these graphs the load was applied progressively until failure of the beam occurred. For the two lower series of graphs of Figures 26 and 27 the letters designating the location of cracks are found in Figures 24(*a*) and 25(*a*), respectively.

In Figures 30 and 31 the stresses in the vertical and horizontal bars of the web are shown in a manner similar to that used in Figures 26 and 27 for the crack widths. The locations of the gauge lines, indicated by numbers on the graphs, are shown by corresponding numbers

in Figures 24 and 25. Only representative gauge lines from Figures 24 and 25 are used in Figures 30 and 31.

In Figures 28 and 29 the stresses in the vertical and horizontal web bars at the load of 640,000 pounds are shown for the purpose of comparison of stresses at different places. The distances of the gauge lines from the top of the beam are shown accurately to scale. The abscissas represent the stresses observed and the approximate positions of the gauge lines to the scales shown. To avoid confusion in plotting the stresses the horizontal positions of gauge lines are shown to the nearest 10 inches from the vertical center line of the beam, and

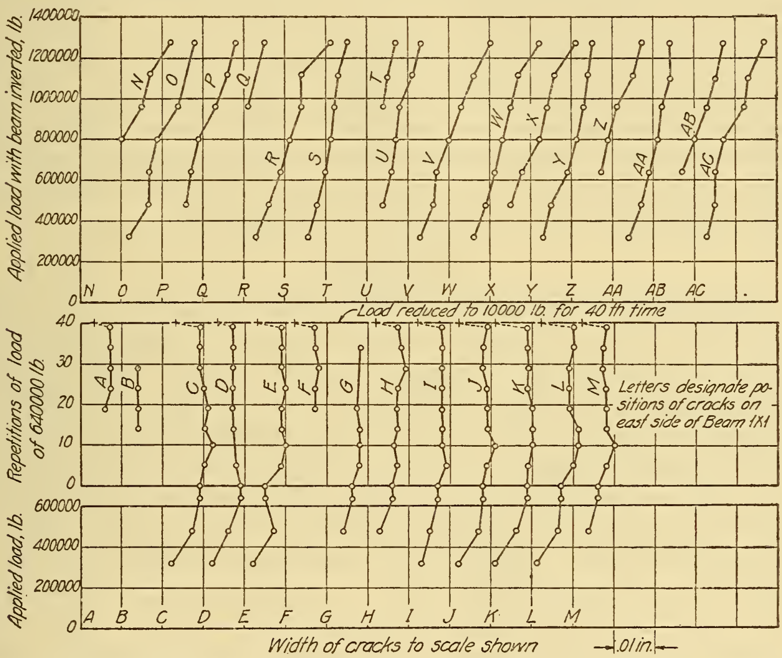

FIG. 27.—*Crack widths measured on east side of beam 1X1 at points marked by letters in Figure 25 (a) and (b)*

the ordinates passing through their center lines are used as the zero axis of stresses. Stresses are plotted horizontally for both vertical and horizontal gauge lines.

4. CRACKS

First crack apppeared at 280,000-pound load, on both faces of beam at north end and in a diagonal direction across the second panel from the center pilaster. Under 40 repetitions of the 640,000-pound load the cracks observed under the first loading extended slightly and only a very few new cracks appeared.

In Figures 26 and 27 the widths of the cracks for various loads and for repetitions of the same load are shown. It will be seen that for

the first few repetitions of the load of 640,000 pounds there was generally a slight tendency toward an increase of the crack width. After about 10 repetitions of the load the crack widths decreased slightly, and after the load had been reduced to 10,000 pounds the cracks closed to a width of about 0.003 inch. A number of new cracks developed during the repetitions of the 640,000-pound load, and it is probable that their formation is the cause of the decrease shown in Figure 26 in the width of cracks which were measured during the repeated loading.

With the beam inverted the first crack on the north end of the beam appeared at a load of 230,000 pounds and that on the south end at a load of 260,000 pounds. Cracks increased in size as the load increased, and were about the same in number as those in the original test and were approximately at right angles with them. This is well shown by the photograph (fig. 25(*b*)). The largest cracks at high loads were in the outer panels at each end of the beam and extended down into the corner at the support point. These were larger and farther apart than those at the point of final failure.

5. FAILURE

For awhile it appeared that failure would take place at the cracks mentioned above. Flaking of the concrete at these places was observed under a load of about 1,320,000 pounds. At 1,330,000 pounds spalling was observed at gauge line 15 (see fig. 24(*b*)), and this spalling increased rapidly, precipitating a sudden failure at a load of 1,363,000 pounds, accompanied by a complete shattering of the web at the center of the center panel on the south end from top to bottom, the failure extending diagonally through the flange toward the load and support points. Failure was accompanied by a very loud report, and the load dropped off at once to 220,000 pounds. Broken concrete from the beam was thrown several feet away. It was subsequently found that all the concrete between the two center vertical rods in the panel where failure occurred was so badly crushed as to be easily removable for the entire depth from top to bottom flange, and the flanges themselves were held together by little except the bare steel.

At failure the stresses in the web steel were still far below the yield point, and the failure was due to crushing and shearing of the concrete. In this beam the stiffener rods (see p. 416) were well anchored and the stiffeners acted effectively to localize the failure to a single panel.

6. STRESSES IN HORIZONTAL BARS

The stresses in the horizontal gauge lines for the reinforcement of the web and the top and bottom flanges at the load of 640,000 pounds are given in Figure 28. It will be seen that the stresses in

the horizontal web bars were nearly equal throughout the depth. It is evident that a plane section before bending did not remain plane after bending. Only the bars in the top flange were in compression. The sum of the tensile stresses in the horizontal bars appears from the diagram in Figure 28 to be greater than the sum of the compressive stresses in the top flange bars, but assuming the strain in the concrete of the top layer

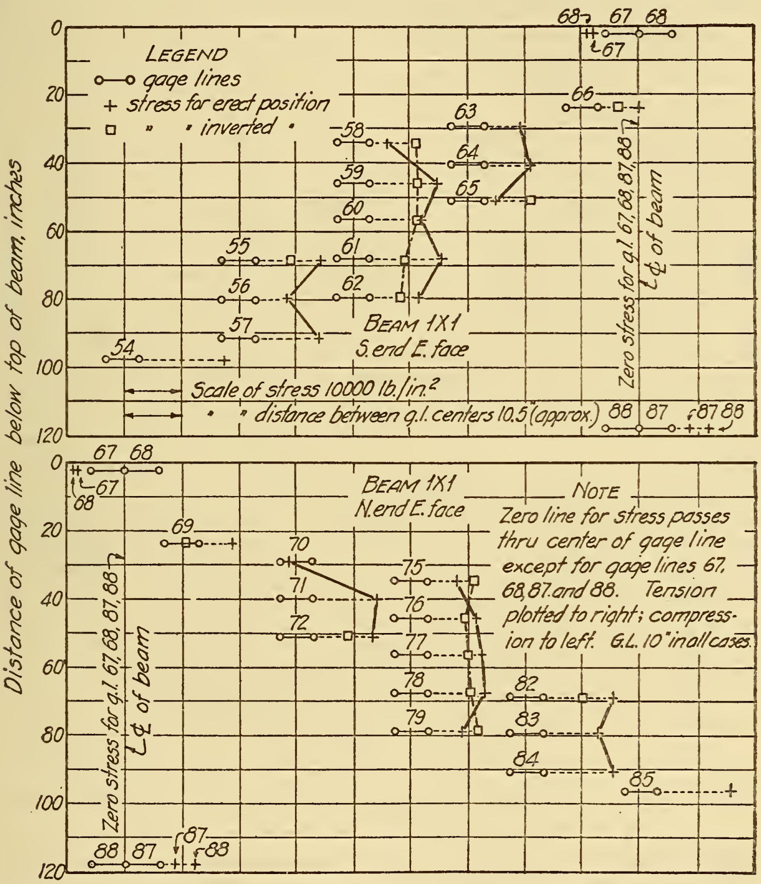

FIG. 28.—*Stresses in horizontal bars of beam 1X1 for gauge lines shown in Figure 25*

of bars to be equal to that measured in the top reinforcing bars, the total horizontal compressive force would be equal to the total horizontal tensile force (neglecting any tension in the concrete) if the neutral axis were about 125 inches below the compression surface. The sum of the moments of the observed tensile stresses in all the horizontal reinforcement about the centroid of the compressive stresses (taken as 5 inches below the compression surface) was 97 per cent of the applied moment under the 640,000-pound load.

7. STRESSES IN VERTICAL BARS

The tensile stresses at various positions in the vertical web reinforcement are shown in Figure 29 for the first application of the 640,000-pound load and for the inverted beam under the same load. Before inverting the beam it had been loaded 40 times with 640,000 pounds, yet the stresses were generally slightly smaller than under the first loading. It is possible that the failure of cracks to close completely after removal of the load held the reinforcing bars under

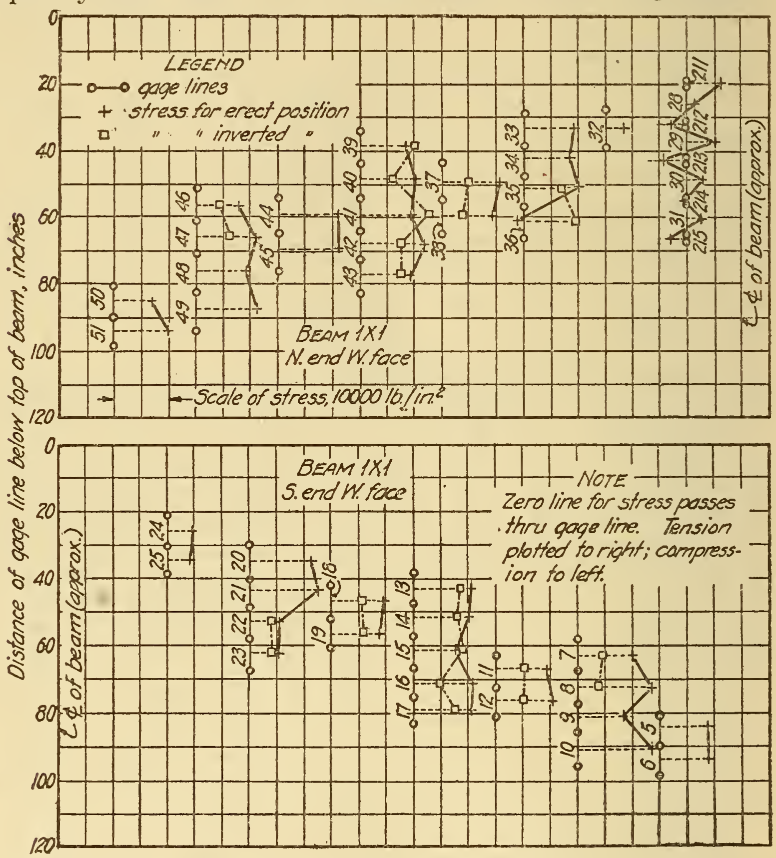

FIG. 29.—*Stresses in vertical web bars of beam 1X1 for gauge lines shown in Figure 24*

some initial strain at the beginning of the test with the beam inverted. This initial strain would not be included in the strain observed under the later test and may be sufficient to account for the decrease in stresses below those observed under the first application of load. Whether unmeasured initial stresses were present or not, at least there is no evidence that the resistance of the beam to web stresses when loaded in the inverted position was any less because of its having been loaded and generally cracked while in the original position.

8. EFFECT OF REPETITION OF LOAD ON STRESSES

At a number of gauge lines the stresses were observed several times during the repetitions of the load of 640,000 pounds. These stresses are plotted in Figures 30 and 31 for all the loads at which strain-gauge readings were taken. The graphs of stress in the web reinforcement are very similar to the graphs of crack widths in Figures 26 and 27. They show, on the whole, that there was very little increase of stress due to the repetition of load, and that the behavior of the beam under load after it was inverted was essentially the same as in the original test in the erect position.

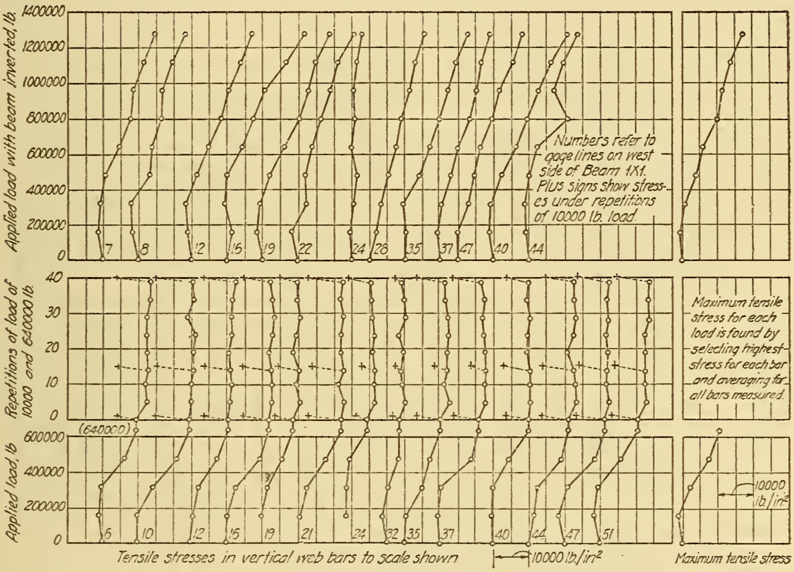

FIG. 30.—*Stresses in vertical web bars for certain gauge lines on west side of beam shown in Figure 24*

9. DISTRIBUTION OF COMPRESSION IN PILASTER AND SHEAR IN WEB

On the face of the center pilaster strains in the concrete were observed in vertical gauge lines for the purpose of ascertaining how the load was distributed into the web of the beam. The results of these measurements are given in Figure 32 for a load of 640,000 pounds. At 15 inches below the top of the beam, gauge lines 201, 202, and 203, it appears that the stress was not uniformly distributed over the section of the pilaster. In gauge line 201 close to the web the stress was about 1,600 lbs./in.^{2} (using the modulus of elasticity of 4,840,000 lbs./in.^{2}), while at the gauge lines 202 and 203 it was only about 450 lbs./in.^{2} At 20 inches below the top of the beam; that is, in gauge lines 101, 102, and 103, the stress seems to have been nearly uniform over the section at about 500 lbs./in.^{2}

From gauge line 101 (20 inches below the top of the beam) downward Figure 32 indicates a general tendency toward a decrease in compressive stress, though the variation is quite irregular. At gauge line 207, about 37 inches below the top of the beam, the pilaster appears to have been in tension. Gauge line 29 on a vertical web bar was very close to gauge line 207 (see fig. 24 (*a*), sec. *A–A*), and it will be seen in Figure 29 that gauge line 29 was in tension, whereas compression existed immediately above and immediately below it. The fact that web cracks extended close to the row of gauge lines

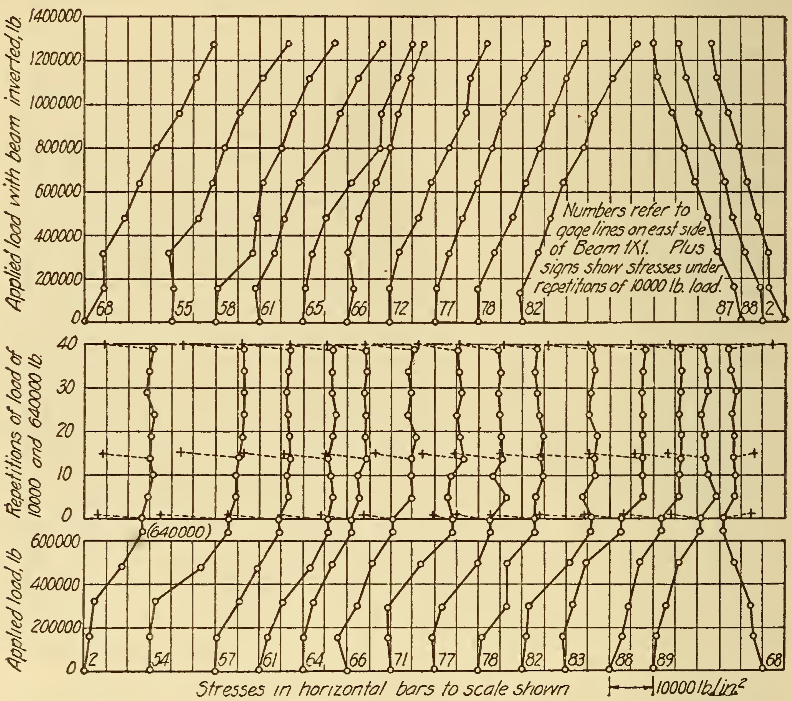

FIG. 31.—*Stresses in horizontal bars of beam 1X1 for certain gauge lines shown in Figure 25*

marked "Group 1" in Figure 32 may help to explain these erratic readings.

The average stress of 500 lbs./in.² at a depth of 20 inches below the top of the beam accounts for a total load of 294,000 pounds, or less than half the load on the beam. If with a total load of 640,000 pounds on the beam the load carried by the pilaster at a depth of 20 inches was only 295,000 pounds, then 345,000 pounds must have been transferred to the webs as shear in the depth of 20 inches. There were no diagonal cracks in the flanges, and therefore it is not likely that the shearing stress in them was greater than 200 lbs./in.² Under these conditions the average shearing stress in the web above

the section referred to would have to be about 2,500 lbs./in.² While it would not be impossible for the web to carry a shearing stress of this magnitude, it seems improbable that it did so at this load because (1) under the maximum load of 1,363,000 pounds on the beam the shearing stress would be over 5,000 lbs./in.² at a depth of 20 inches below the top of the beam, and (2) the decrease in shearing stress below that section would have to be so rapid, in order that the average should not be greater than the remaining load divided by the shearing area available (375 lbs./in.²), that near the bottom of the beam the shearing stress would probably be less than 100 lbs./in.² Figure 24 (*a*) and (*b*) shows that the diagonal cracks existed

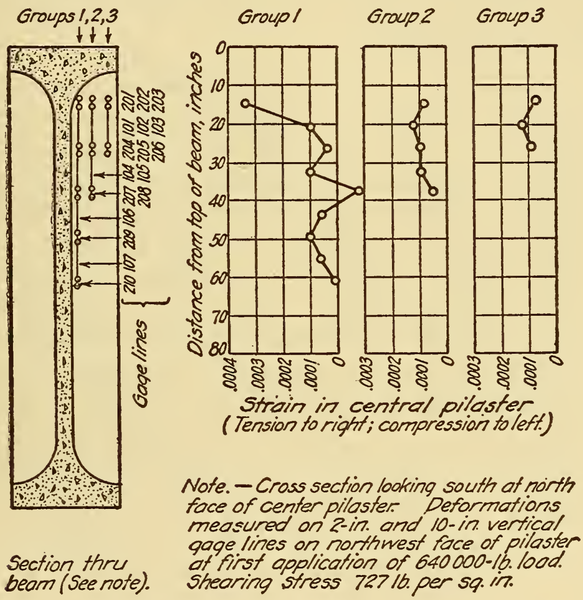

FIG. 32.—*Vertical strain measured in central pilaster of beam 1X1*

close to the center pilaster throughout the depth of the beam, so the shearing stress must have been over 100 lbs./in.² throughout the depth. The cracks near the bottom were, however, much farther apart than those near the top, so it seems that the measurements of compressive strains in the pilaster gave a correct idea of the manner of distribution of shearing stresses, though the intensity at any point in the depth is subject to question. The strains shown in Figure 32 were found in only one-quarter of the pilaster. It is possible that the average over an entire section of the pilaster, if known, would be consistent with the possibilities of the web for resisting shearing stresses.

XI. EFFECT OF STRENGTH OF CONCRETE ON TENSILE STRESSES IN WEB REINFORCEMENT AND ON ULTIMATE STRENGTH OF BEAMS

In order to determine the effect of variation in the quality of the concrete on the tensile stresses developed in the web reinforcement and on the behavior of the beams in other respects seven groups of beams were made, in each of which the only variable introduced was the richness of the concrete. Groups representing four percentages of vertical web reinforcement and three percentages of diagonal web reinforcement were included. All the beams had a nominal web thickness of 3 inches and a total depth of 36 inches. From Table 3 the laboratory numbers of the beams included in these groups may be found.

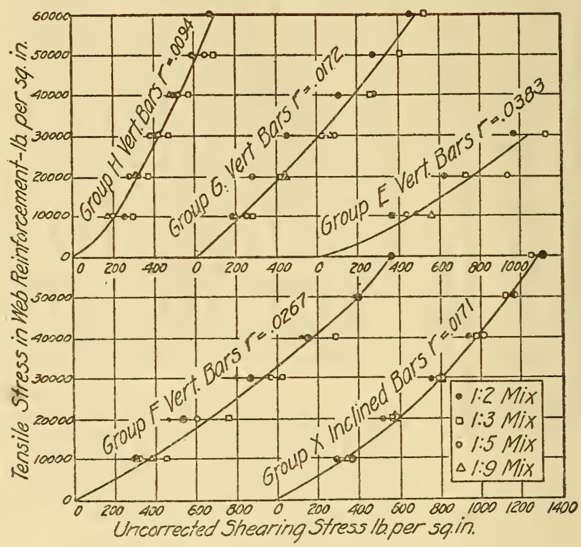

FIG. 33.—*Relation between shearing stress and tension in web reinforcement for concretes of varying strengths*

For the five groups of beams having varying percentages of vertical and of inclined web reinforcement Figure 33 shows the relation between the shearing and the tensile stresses for the various mixes used. The legend shows the mix. The strength of the concrete as determined by tests of 8 by 16 inch control cylinders is given in Table 1. The actual web thickness is given in Table 7.

Each of the curves in Figure 33 indicates that for any given shear the tensile stress is substantially the same, regardless of the compressive strength of the concrete. In many cases the beams with the strongest concrete showed the highest tensile stress at any given shearing stress. Those with the next strongest concrete frequently

showed the lowest tensile stresses at any given shearing stress. This indicates that the variations in the tensile stress at a given shear were due to accidental causes, and not to variation in the strength of the concrete.

Figures 38, 39, 40, and 41, which contain the data of all the beams of these groups and which show by distinct characters the various mixes of concrete, indicate that it is not necessary to introduce into the equation between tensile stress and shearing stress a term which takes account of variations in strength of the concrete.

Although, as brought out in the preceding paragraphs, the tensile stresses were independent of the compressive strength of the concrete, the ultimate strength of the beam may be dependent upon the strength of the concrete. For the beams with vertical bars, shown in Figure 33, the highest shearing stresses shown are 700 lbs./in.^{2} for the 1 to 9 mix, 950 lbs./in.^{2} for the 1 to 5 mix, 1,300 lbs./in.^{2} for the 1 to 3 mix, and 1,550 lbs./in.^{2} for the 1 to 2 mix. With the leaner mixes failure, apparently by crushing of the web concrete, took place before the web reinforcement was stressed to the yield point. In Table 7 the cause of failure for the beams with lean concrete is given as vertical shear. As indicated in Section IX, page 416, however, a vertical shear failure is very intimately related to a diagonal compression failure. For the 1 to 2 mix the failure was by tension in the web reinforcement. Likewise for beams with inclined bars the points representing the weaker concretes drop out of the diagram at shearing stresses lower than those which correspond to the yield-point stress in the web reinforcement. These tests indicate clearly that with efficient distribution and anchorage of longitudinal reinforcement and of web reinforcement failure may occur due to the weakness of the concrete of the web in compression. As the amount of web reinforcement is increased above a certain limit a corresponding increase in the strength of the concrete must be secured in order to avoid diagonal compression failure. Analysis indicates (Sec. II, equation (19), p. 396) that the limit for the amount of tension web reinforcement which is allowable is reached when the shearing stress is equal to the compressive strength of the concrete for beams with inclined (45°) web reinforcement, or when it is equal to one-half the compressive strength of the concrete for beams with vertical web reinforcement. The test results are in accord with the analysis in that the beams with diagonal web reinforcement carried larger shearing stresses without producing diagonal compression failure than did beams with vertical web reinforcement. With the beams having large amounts of vertical web reinforcement, diagonal compression failure occurred when the shearing stress was less than half the compressive strength of the concrete.

XII. WEB STIFFENERS

In series 1 all the beams except 1C1 had stiffeners of concrete cast as a part of the web. These stiffeners were employed to simulate the effect of frames in the concrete ships in order to determine whether the frames performed any function in resisting the shearing stresses. Four of the beams had stiffeners on one side only, and nine had stiffeners on both sides. The size and spacing of the stiffeners are designated in Table 2 under the caption "Longitudinal section and side elevation." The letters in this column refer to Figure 5. The form of cross section between stiffeners is shown in Figure 6, *A* and *B*.

Beams 1K1 and 1L1 had two three-fourths inch round bars in each stiffener. Beam 1X1 had four one-half inch round bars, and all other beams of series 1 had four three-fourths inch round bars in each stiffener. In beam 1X1 the stiffener rods were hooked around the longitudinal bars in the top and in the bottom of the beam. In all other beams they had no anchorage except by bond.

A study of the test results indicated that the stiffeners had some effect in causing the cracks to take a direction somewhat more nearly vertical than was the case with beam 1C1, which differed from the others in that it had no stiffeners. In the case of beam 1X1 the failure was by vertical shear and was confined entirely to the space between two stiffeners. (See fig. 25(*b*).) The stiffeners probably played an important part in localizing the failure. They seemed also to have the effect of restricting slightly the size of cracks. The effect on the maximum load carried was not distinct, but there seemed to be a slight addition of strength due to the presence of the stiffeners.

XIII. VARIATION IN WEB THICKNESS

Equation (17), Section II, is found to reduce to the form

$$v = rf_v \tag{27}$$

for beams with either vertical or 45° web reinforcement; that is for $\alpha = 90°$ or 45°. It will be seen that with a given size of bar and a given spacing of stirrups the thinner the web the greater will be the shearing stress, as given by this equation, for any given tensile stress. When the web thickness becomes zero the ratio, r, becomes infinite, and nominally the shearing stress would become infinite also. From this it might appear that a beam having steel, but no concrete, in the web would, according to the equation, carry an infinite load. That this interpretation of the equation is incorrect will be apparent if an attempt be made to compute the total load from equation (27) for a zero web thickness. The total load is $W = 2vbjd$. In this expression b is zero, v is infinite, and the product is indeterminate. Therefore this equation can not be used to determine the total load for such a

case. The equation may, however, be put into a form which is better suited to studying the effect of varying web thickness.

Multiplying both sides of the equation (27) by b' gives

$$b'v = b'rf_v \tag{28}$$

from which

$$\frac{V}{jd} = \frac{b'A}{ab'}f_v = \frac{A}{a}f_v \tag{29}$$

since

$$b'v = \frac{V}{jd} \text{ and } r = \frac{A}{ab'},$$

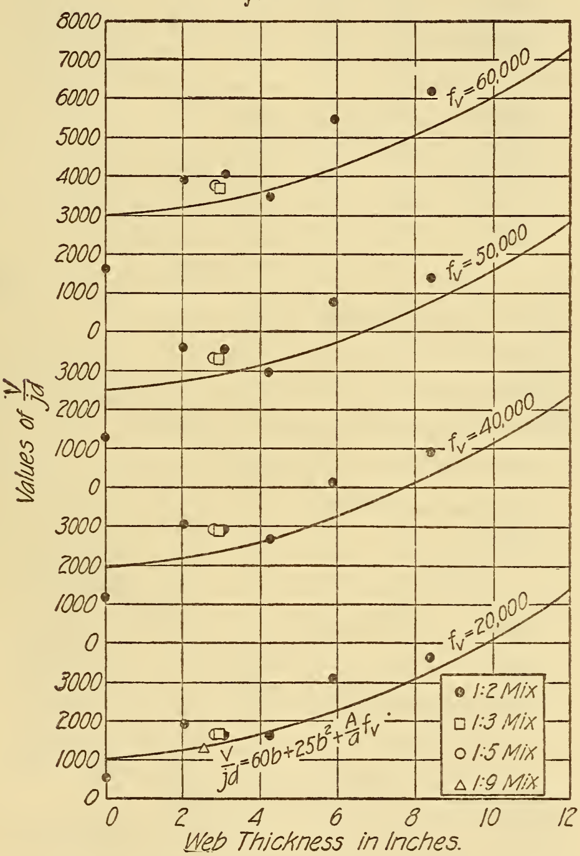

FIG. 34.—*Shearing force,* $\frac{V}{jd}$, *per unit of length for beams with one-half-inch inclined bars*

The form of this equation would indicate that the entire load is carried due to the development of tensile stress in the web reinforcement. The web thickness b' does not appear in this equation, and it would seem that the load should be independent of web thickness. This should be expected from the analysis, since all the tensile stresses were assumed to be carried by the web reinforcement.

In Figure 34 web thicknesses are plotted as abscissas, and values of $\frac{V}{jd}$ for tensile stresses in the web reinforcement of 20,000, 40,000, 50,000, and 60,000 lbs./in.² are plotted as ordinates. If, as is indicated by equation (29), the concrete were not at all effective in resisting the diagonal tension, the strengths, $\frac{V}{jd}$, of the beams would be represented by a horizontal line intersecting the curves of Figure 34

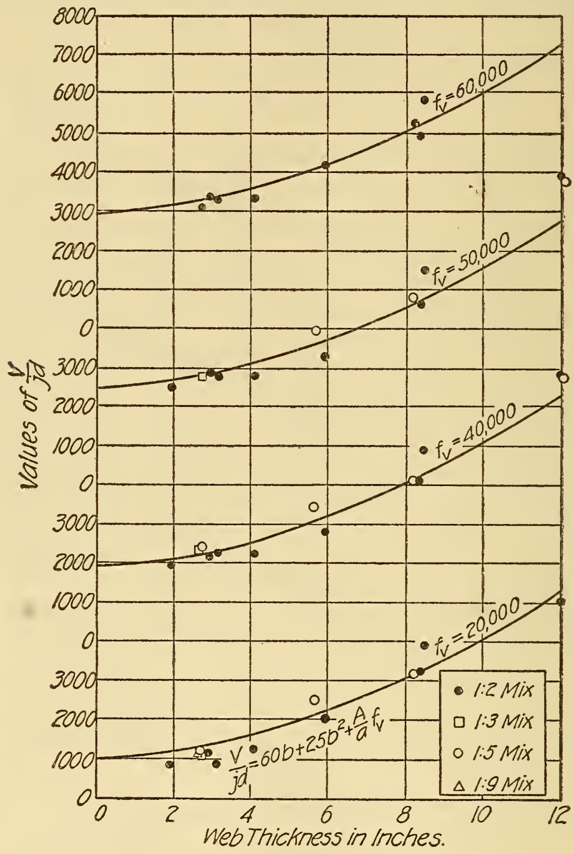

FIG. 35.—*Shearing force,* $\frac{V}{jd}$, *per unit of length for beams with one-half-inch vertical stirrups*

at points where $b'=0$. If the concrete were effective at a shearing unit stress of a fixed amount, $\frac{V}{jd}$ would be represented by an inclined straight line passing through the same intersections. An effort was made to fit an inclined straight line to these points, but any straight line which fitted these data would not fit the data in Figures 38 and 39. The parabola having the equation

$$\frac{V}{jd}=60b'+25b'^2+\frac{A}{a}f_v \tag{30}$$

fits the data of both sets of figures fairly well and with about equal precision. This equation also fits the data of the beams with vertical web reinforcement as shown in Figures 35, 36, and 37.

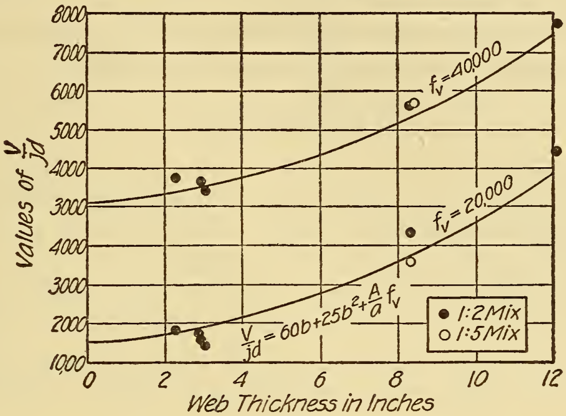

FIG. 36.—*Shearing force,* $\frac{V}{jd}$*, per unit of length for beams with five-eighths-inch vertical stirrups*

The point at which b' equals zero represents the strength which a beam of zero web thickness should have if the analysis applied to such a case. Since one of the assumptions underlying the analysis is that there is a double system of web members, one resisting tension and the other resisting compression, it is clear that if the compression system is absent the analysis can not be expected to apply. In spite of the absence of the compression system the beams with

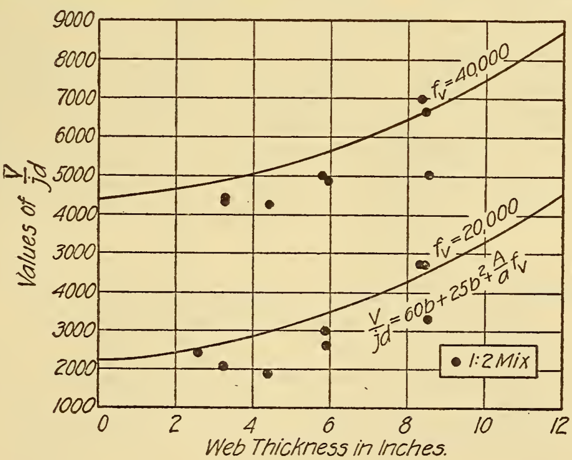

FIG. 37.—*Shearing force,* $\frac{V}{jd}$*, per unit of length for beams with three-fourths-inch vertical stirrups*

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one-half inch tension diagonals but with no concrete in the web carried a considerable load, as is shown in Figure 34. By resisting the horizontal shear the end pilasters to some extent take the place of the diagonal compression members.

If equation (30) be reduced to terms of shearing unit stress by dividing through by the web thickness b' it becomes

$$v = 60 + 25b' + rf_v \tag{31}$$

As brought out in the previous discussion, the last term in this expression represents the part played by the web reinforcement in adding to the strength of the beam. The remainder of the second term must, therefore, represent the part played by the concrete of the web in adding to the strength of the beam. Obviously, the concrete will not be more effective than would the same concrete in a beam having no web reinforcement, and this portion of the strength can not be expected to increase indefinitely with increase in the web thickness.

This study will help to remove any impression that the thinness of the webs used in the majority of the beams may have been responsible for the high shearing unit stresses developed. The results show, on the contrary, that the effectiveness of the concrete increased with increasing web thickness. With the very thin webs the reinforcement occupied a considerable portion of the thickness of the web and probably destroyed to some extent the effectiveness of the concrete which was present. This is brought out by the fact that for the beams having horizontal rods in the web failure was generally by horizontal shear. With an increasing web thickness this destructive effect would be somewhat smaller proportionally, and after the point is reached beyond which the web thickness is large in proportion to the width occupied by the web reinforcement the effectiveness of the concrete probably would not increase with further increase in web thickness.

XIV. RELATION BETWEEN SHEAR IN WEB AND TENSILE STRESS IN WEB REINFORCEMENT

In Figures 38 and 39 the ordinates are calculated as stated in Section VI, page 409, and corrected for stiffness of frames by the method given in Section VII, page 412. The abscissas are ratios $\left(r = \frac{A}{bs \sin a}\right)$ of web reinforcement to web concrete. See analysis in Section II, page 392. The shearing stresses are those for loads which gave observed tensile stresses in the web reinforcement of 20,000, 40,000, 50,000, and 60,000 lbs./in.^{2}, respectively. Similarly, Figures 40 and 41 show uncorrected shearing stresses.

Figure 38 includes the data of all 36-inch beams having vertical stirrups except those having also horizontal or inclined bars as web reinforcement, and except those having no concrete web. In addition to the corrected shearing stresses for the 36-inch beams, Figure 38 contains the uncorrected shearing stresses for the 18-inch beam 4BE21 and for a group of beams 15.7 inches deep tested at the materials testing laboratory of the Royal Technical High School at Stuttgart, Germany.[4] Since beam 4BE21 was rectangular in cross sec-

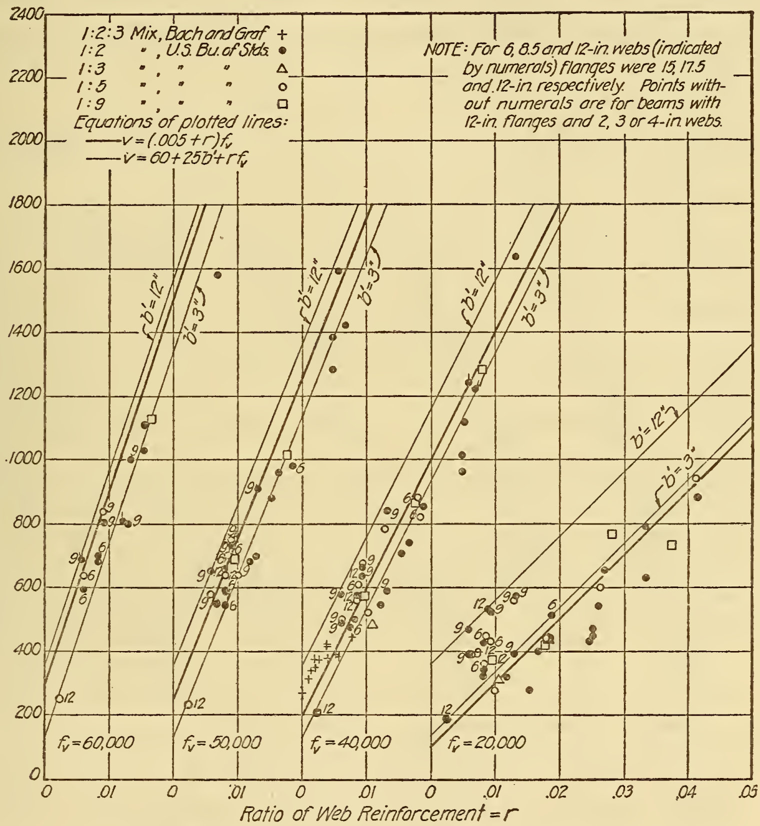

FIG. 38.—*Corrected shearing stress in vertically reinforced beams for various tensile stresses and ratios of web reinforcement*

tion, no correction for frame strength was necessary. The beams from the Stuttgart laboratory were T-shaped and of such slenderness that a correction for frame strength does not seem necessary.

Figure 39 includes the data of all 36-inch beams having inclined tension reinforcement in the web, except those having also vertical stirrups and those having no concrete web. The beams with ex-

[4] C. Bach and O. Graf, "Versuche mit Eisenbeton-Balken zur Ermittlung der Widerstandsfähigkeit verschiedener Bewehrung Gegen Schubkräfte," Deutscher Ausschuss für Eisenbeton, Heft 10.

panded metal web reinforcement are included, as are also those with loose bars in the web inclined at 45° in the compression direction.

The corrections for stiffness of frame were based upon test results of beams with no webs (strictly speaking, frames). As there were no such frames for any except the 36-inch beams, there was no means of applying corrections to the beams of 120, 52, 48, and 18 inch depths. These beams were, therefore, omitted from Figures 38 and 39 (except beam 4BE21, which was rectangular and needed no correction), but they are included in Figures 40 and 41, as well as the uncorrected shearing stresses for the beams included in Figures 38 and 39.

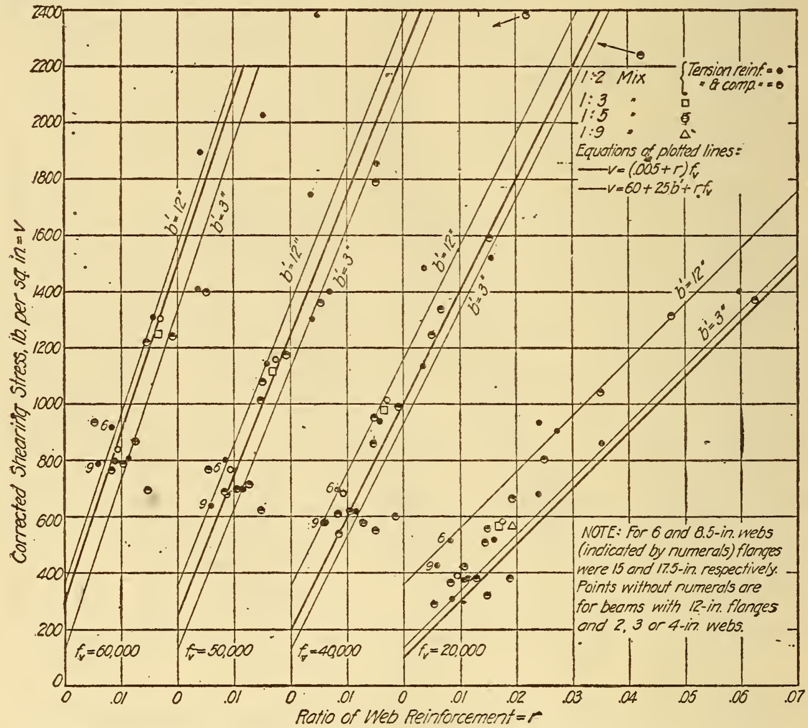

FIG. 39.—*Corrected shearing stress in diagonally reinforced beams for various tensile stresses and ratios of web reinforcement*

For beams having web reinforcement it has been brought out in a previous discussion that the effectiveness of the concrete in the web of the beam in resisting shear increases as the web thickness increases. The equation which fits the data of the beams with varying web thicknesses (see equation (32) and fig. 34) shows so great an increase in effectiveness of the concrete with increasing web thickness that it is unreasonable to expect this relation to continue indefinitely. The equation

$$v = (0.005 + r)f_v \tag{32}$$

fits the data of Figures 38 and 39 almost as well as equation (31),

but it does not fit Figures 34, 35, 36, and 37 because the web thickness does not enter into this equation in the same degree as it enters into equation (31). Equation (32) assumes that the effectiveness of the concrete of the web increases as the tensile stress in the web reinforcement increases, but that it is independent of the web thickness. Equation (31) states that the effectiveness of the concrete is proportional to the web thickness, but that it is independent of the tensile stress in the web reinforcement. Equation (32) fits the data reasonably well and is more conservative in its interpretation of the strength of beams having web thicknesses greater than those represented in these tests. Also for even the highest stresses it does not attribute greater effectiveness to the concrete of the webs than was found to exist in these tests. For these reasons an equation of this form seems more logical for use in estimating the shearing strength of a beam than equation (31), which was introduced to harmonize with other data of the tests, the data which show the effect of web thickness.

Since in Figures 38 and 39 the shearing stresses plotted as ordinates have been corrected for the strength of the frame, those shearing stresses may be taken as the stresses to be developed in rectangular beams when the tensile stresses in the web reinforcement are as given in those figures. The fact that the points for the rectangular beams (having 12-inch webs) fall well above the average for all the points tends to confirm the reasoning which indicates that the data of Figures 38 and 39 apply to rectangular beams. Attention is called, however, to the fact that the tensile stresses in the web reinforcement of beam 4BE21, which is a rectangular beam 18 inches deep and having a web thickness of 12 inches, were greater than those given by equation (32). The data for this beam have been plotted in Figure 49. The test results are discussed in Section XX, page 451.

On the whole, equation (32) appears to represent fairly well the relation between the shearing stress and the tensile stresses developed in closely spaced stirrups placed at either 45 or 90° with the axis of the beam. The manner in which this relation is affected by the spacing of the stirrups is discussed in Section XIX, page 450, but was not fully determined.

In Figures 40 and 41 the uncorrected shearing stresses have been plotted as ordinates; that is, the shearing unit stresses in these figures represent the total shearing strengths of the beams at the corresponding tensile stresses regardless of whether the strength comes from the web alone or is added to by the shearing stresses in the heavy frames. Graphs of equation (32) are shown in Figures 40 and 41 in order to facilitate a comparison with the corrected shearing stresses of Figures 38 and 39. While for beams having flanges so large as to constitute a frame some shearing strength is

available above that afforded by the relatively thin webs, there seems to be too much uncertainty in the evaluation of the additional strength to warrant the proposal of a method of using it.

The shearing strengths of beams with inclined web reinforcement are seen in Figures 38, 39, 40, and 41 to have been slightly greater than those of beams with vertical web reinforcement. The difference is too small, however, to warrant the proposal of different methods for computing the shearing strengths for the two cases.

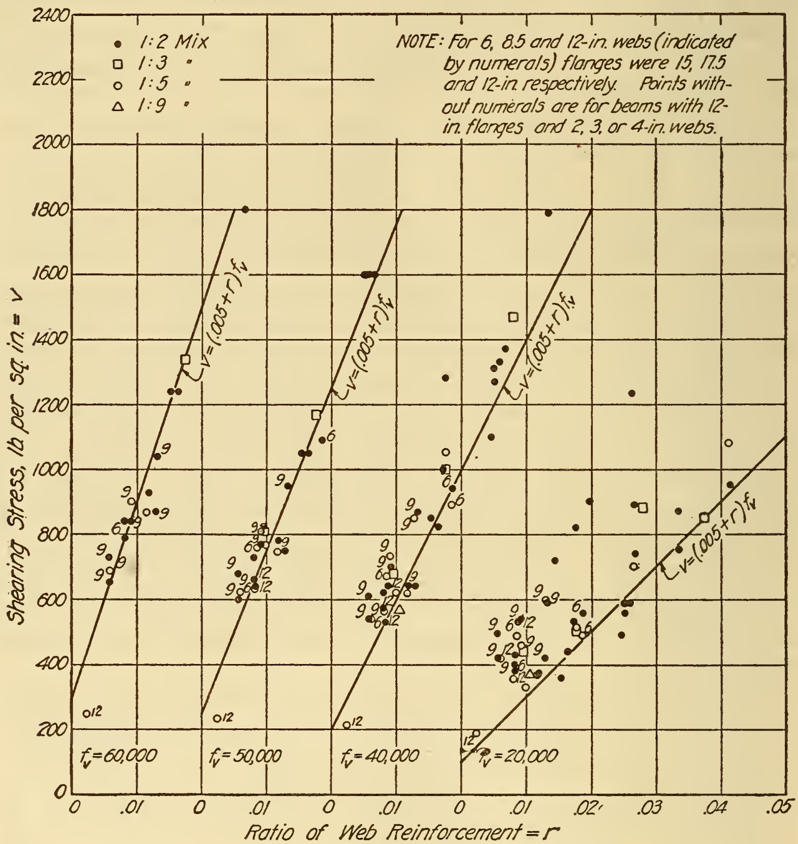

FIG. 40.—*Uncorrected shearing stress in vertically reinforced beams for various tensile stresses and ratios of web reinforcement*

XV. HORIZONTAL WEB REINFORCEMENT

Horizontal bars were used in the webs of a number of beams. The numbers of these beams are shown in Tables 1, 3, and 7. All the beams were 36 inches deep and had 12-inch flanges, with eight 1¼-inch bars in the top and eight in the bottom flange. Concrete of a 1:2 mix was used in all of them.

These beams were included in the investigation, not because it was expected that horizontal bars in the web would strengthen the beam, but because the shell of the concrete ship was designed with horizonal bars in it and it became important to determine the effect of such bars on the behavior of a beam.

From the strain-gauge measurements it was found that the tensile stress in the vertical stirrups was less for the beams which had horizontal web reinforcement than for those which had none. The decrease in the stress in the stirrups was approximately 15 per cent for each 1 per cent of horizontal web reinforcement. The reason

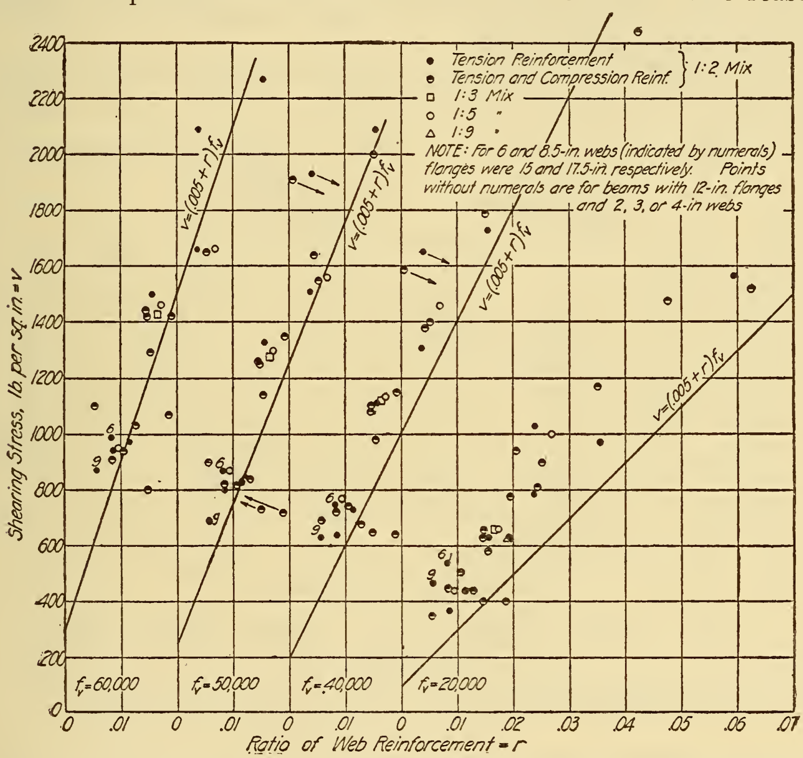

FIG. 41.—*Uncorrected shearing stress in diagonally reinforced beams for various tensile stresses and ratios of web reinforcement*

for this effect of the horizontal reinforcement is not clear. The advantage of it was lost, however, due to the fact that the concrete web was so reduced in section that there was a tendency for failure to occur in the plane of the horizontal reinforcement. Although the failure probably was due primarily to the development of high diagonal tension and diagonal compression at this plane, these diagonal stresses accompanied high horizontal shearing stresses. Marked horizontal detrusion of the parts above and below the plane of failure occurred. Figure 19, a photograph of beam 4B2, shows a pronounced failure by horizontal shear.

In Table 8 the total loads carried by beams having horizontal web reinforcement are shown, and in Table 7 the cause of failure. In general, it may be said that the strength of the beam decreased as the diameter of the horizontal bars increased. All beams with 3-inch webs and horizontal web bars failed by horizontal shear. Of the beams having 4-inch webs, only two having horizontal web bars did not fail by horizontal shear. The average load for these two beams was almost the same as that for the similar beams having no horizontal web reinforcement. All of those which failed by horizontal shear failed under less load than that carried by a similar beam without horizontal web reinforcement.

On the whole, little can be said in favor of using horizontal bars in the web of a beam. Sometimes, however, horizontal bars will be

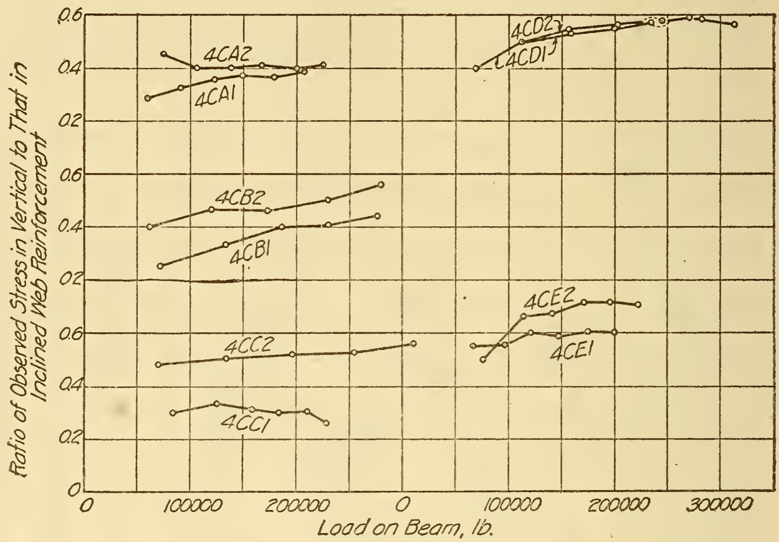

FIG. 42.—*Ratio of stress in vertical to that in diagonal tension reinforcement at different loads for beams with three-way reinforcement*

present and the data will give a basis for determining the lower limits of web thickness which must be used.

XVI. COMBINED VERTICAL AND INCLINED WEB REINFORCEMENT

A group of beams was made and tested in which vertical bars, bars inclined at 45° in the direction of the diagonal tension and at 45° in the direction of diagonal compression, were present all in the same beam. The numbers of the beams are given in Tables 1 and 3.

It is pointed out (in Sec. XXIV, p. 468) that for a given stress in the web reinforcement the deflection of the beams with vertical web bars was greater than that of the beams with inclined bars. This

would lead to the expectation that the equation (32), which applies about equally well to beams with vertical web bars and to those with inclined web bars, would not apply to those with both vertical and inclined web bars. Since it was found from the beams with two-way inclined bars that the tensile stress in the web bars was independent of the presence of compression reinforcement in the web, it is assumed that the same would be true when vertical bars were present also, and the relation between the tensile stresses in the diagonal tension and the vertical bars has been studied without considering the diagonal compression bars.

It was found that for a given load the tensile stress in the vertical bars was in all cases less than that in the inclined tension bars of the same beam. The ratio of these stresses is shown in Figure 42 for all loads for all the beams having both sets of bars. In Figure 43 the average ratios for the five groups are plotted as ordinates and the sum

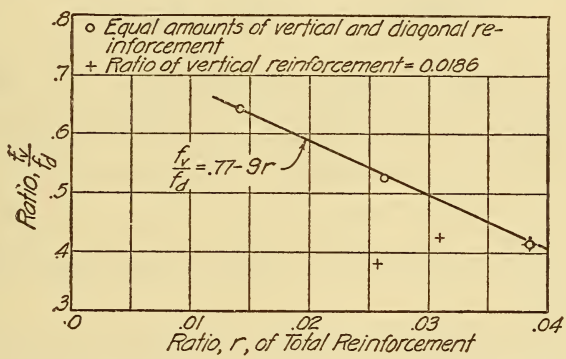

FIG. 43.—*Average ratios of stress in vertical to stress in diagonal tension reinforcement*

of the ratios of inclined and vertical web reinforcement as abscissas. From these data an average ratio of tensile stress, f_v, in the vertical bars to the tensile stress, f_d, in the inclined bars of the same beam has been expressed in the form of the equation

$$\frac{f_v}{f_d} = 0.77 - 9r \tag{33}$$

in which r is the sum of the ratios of vertical and of diagonal web reinforcement. As will be seen from the diagram, this equation applies only to the beams having equal amounts of vertical and diagonal reinforcement. Equation (32) may be taken as stating that in a beam with tension reinforcement in the web in one direction only the shearing resistance v comes from two sources, (1) the concrete represented by the term $0.005\ f_v$, and (2) the web reinforcement represented by the term rf_v. For a beam with both vertical and inclined reinforcement the same equation might be used if the stresses

in the two sets of reinforcement were the same, since equation (32) was found to apply reasonably well to both vertical and inclined reinforcement. Since the stresses in the two systems of reinforcement are different, an equation analogous to (32) may be derived by using the stresses separately for the two systems. That is,

$$v = 0.005\, f_d + r_v f_v + r_d f_d \qquad (34)$$

For beams with equal amounts of vertical and diagonal web reinforcement $r_v = r_d = \frac{r}{2}$, and from equation (33)

$$f_v = (0.77 - 9r) f_d$$

Therefore

$$v = \left(0.005 + \frac{r}{2}(1.77 - 9r)\right) f_d \qquad (35)$$

or

$$v = \left(0.005 + r_d + r_v(0.77 - 18r_d)\right) f_d \qquad (35a)$$

The graphs of equation (35a) for tensile stresses of 20,000, 40,000, 50,000, and 60,000 lbs./in.^{2} are shown in Figure 44, also the points representing the shearing and tensile stresses in the beams with both vertical and inclined reinforcement. The agreement between the graphs and the location of the points is fair, but not much better than the agreement of the points with the graphs of equation (32), in which r is used as the sum of the ratios of the vertical and inclined reinforcement.

Figure 45 has been prepared to afford a comparison of the corrected shearing stress at maximum load for the beams having either vertical or inclined tension reinforcement in the web with the shearing stress for the beams having both systems of reinforcement. This figure shows that a beam having either vertical or inclined tension reinforcement carried, in general, the same maximum shearing stress as a beam having the same total percentage of both vertical and inclined tension reinforcement. The probable reason for this equality in strength is that after the inclined stirrups were stressed to their yield point the stress in the vertical stirrups increased rapidly, and before the maximum load was reached the two systems had about the same intensity of stress. If this was the case equation (32) should apply to these beams as well as to those with either vertical or diagonal reinforcement, by using r as the sum of the ratios of vertical and diagonal reinforcement.

It is seen that the shearing strength of a beam with a given ratio of web reinforcement, though divided between vertical and diagonal, was in general the same as that of a beam having the same ratio of either vertical or diagonal web reinforcement.

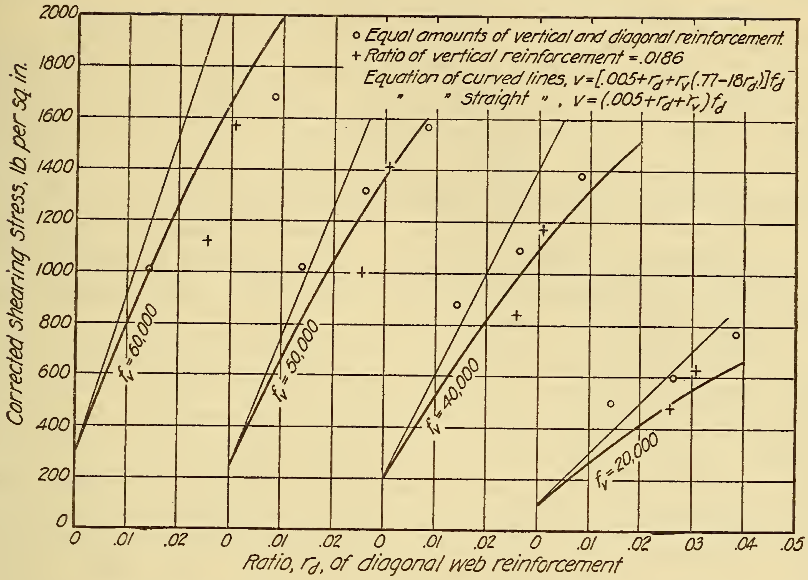

FIG. 44.—*Corrected shearing stress in beams with three-way reinforcement for various tensile stresses and ratios of web reinforcement*

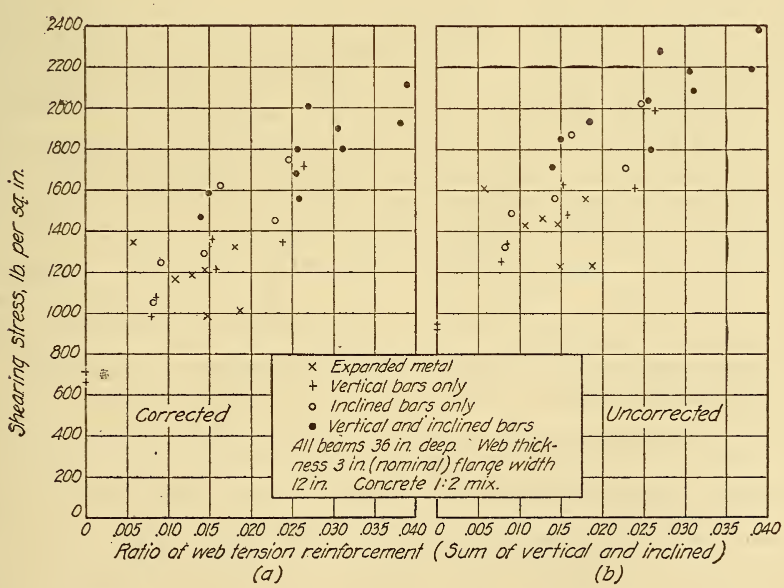

FIG. 45.—*Shearing stresses at maximum load for beams with vertical and inclined web bars and beams with expanded metal web reinforcement*

(*a*) Corrected shearing stresses. (*b*) Uncorrected clearing stresses.

XVII. EXPANDED METAL WEB REINFORCEMENT

Seven beams, 4XA1 to 4XG1, inclusive, used diamond mesh expanded metal as web reinforcement. All of these beams were 36 inches deep, the webs were nominally 3 inches thick, and the flanges were 12 inches wide. Eight 1¼-inch round bars formed the horizontal reinforcement in the top flange and eight in the bottom flange. The expanded metal was placed in the beam with the major axis of the diamond extending vertically. The strands of the metal made an angle with the vertical of about 20 to 22°. The axes of the diamonds were about 3 and 8 inches.

In beam 4XF1 no anchorage of the expanded metal to the 1¼-inch bars in the top and bottom flanges was provided. (See fig. 9 (*L*).) In all other beams the metal was hooked around the top and the bottom bars. (See fig. 9 (*K*).) For all beams the web reinforcement was spliced at the center of the span by lapping one diamond of one sheet over one diamond of the other in the direction of the 3-inch axis. (See fig. 7 (*G*), (*H*), and (*I*).) With this lap three bridges (the portions within which the two strands of a sheet intersect) of each strand of either sheet were interlocked with three bridges of the other sheet. This arrangement gave a lap of about 9 inches measured along the strand. Beam 4XD1 had a similar one-diamond lap at each one-fourth point of the span. (See fig. 7 (*H*).) In beam 4XE1 there was a horizontal lap splice at mid depth of the beam. (See fig. 7 (*I*).) This lap extended in the direction of the 8-inch axis, but was only 5 inches in length and included one bridge and less than one diamond of each sheet. In beams 4XA1, 4XB1, 4XC1, and 4XG1 the reinforcing metal was spliced by lapping only at the center of the span, where a poor joint should not affect the strength of the beam. These beams are spoken of in the discussion as being without splices in the web reinforcement. A single sheet of expanded metal was not sufficient generally to give the necessary web reinforcement. Beam 4XA1 had a single sheet and beam 4XG1 had four parallel sheets in each half span of the beam. All the other beams had two parallel sheets throughout (four at laps).

Assuming the weight of steel as 0.284 lb./in.³, the volume of steel in 1 square foot of web will be $\frac{w}{0.284}$ cubic inch, where w is the weight in pounds of expanded metal per square foot of web, and the volume ratio of web reinforcement is $\frac{w}{0.284 \times 144b'} = \frac{w}{40.9b'}$, where b' is the web thickness in inches. Only half the strands are inclined in the proper direction to take tension. The strain-gauge readings showed clearly that those strands took tension and that those inclined in the

compression direction took compression. The ratio of diagonal tension reinforcement may then be expressed as

$$r=\frac{1}{2}\frac{w}{40.9b'}=\frac{w}{81.8b'} \qquad (36)$$

The ratios of web reinforcement given in Table 7 were computed according to equation (36).

The data of the beams using expanded metal web reinforcement are included in Figures 39 and 40. The indication is that for beams having no splices of the web reinforcement the relation between the shearing stress and the tensile stress in the web reinforcement may be expressed by equation (32), in which r is the ratio of tension reinforcement. The beams having splices developed lower shearing stress at a given tensile stress than did the other beams. Since only half of the metal is available as tension reinforcement, and since diagonal compression reinforcement is not generally necessary, it seems that the use of expanded metal for web reinforcement would not, in general, be an economical use of material.

A comparison of the observed compressive stresses in the expanded metal web reinforcement of beams 4XC1, 4XA1, and 4XG1 with the stresses in the diagonal compression reinforcement of beams 4CA1–2, 4CB1–2, and 4CC1–2 in the order given shows that the expanded metal received only about half as great compressive stress as did bars placed at 45° with the horizontal. This is shown in Figure 48. For beams compared with each other the ratio of compression reinforcement was about equal, and it seems that the difference in slope was responsible for the difference in stress. The greater dependence of the compressive than the tensile stresses upon the slope of the web members is probably due to the fact that in compression the concrete web is intact and capable of taking compression in the most effective direction. In tension this is not true, owing to the cracking of the concrete web.

Beams 4XD1 and 4XE1 were tested for the purpose of determining whether the laps described on page 444 and Figure 7 was sufficient for an effective splice. Measurements of the movement of one sheet relative to the other were taken with the strain gauge by locating one gauge hole upon one sheet and the other gauge hole upon the other sheet. Immediately adjacent to this was a gauge line with both holes upon the same sheet. Change in length of the latter gauge line indicated strain in the metal, while change in length of the former gauge line included strain in the metal and slip of one sheet past the other. The difference in the movement of the two gauge lines would, therefore, be the slip. No slip greater than 0.001 inch was indicated at any load below the maximum. At the maximum

load a slip of about 0.01 inch was indicated for beam 4XD1 and of about 0.02 inch for 4XE1. It is possible that this slip was the cause of failure, but there is at least an equal chance that weakening of the laps by filling up the section with four thicknesses of expanded metal prevented getting good workmanship at this place. Whatever may have been the cause of failure, the stress in the metal was approximately 40,000 lbs./in.^{2} before any slip greater than 0.001 inch occurred.

Tests to determine effectiveness of laps, reported in Technologic Paper No. 233 [5] of the Bureau of Standards, page 329, more direct than those reported here. There the indication was that a lap of about 1.5 diamonds was necessary in order to produce an effective splice of the expanded metal.

The maximum load carried by the beams reinforced with expanded metal appeared to bear little relation to the total amount of web reinforcement present. Beam 4XC1, having the least web reinforcement, carried the greatest load. This beam had only one thickness of expanded metal in the web. Beam 4XG1 had four thicknesses of metal, and its maximum load was very small in view of the large amount of reinforcement present. It is likely that the interference of the multiple thickness web reinforcement with the proper placing of the concrete is responsible for the weakening of these beams.

XVIII. DIAGONAL COMPRESSION

To determine the effect of variation in the strength of the concrete and in the type and amount of web reinforcement on the compressive stresses developed in a diagonal direction in the web, the ratio of the shearing stress to the diagonal compressive strains was studied. In this study beams with vertical web reinforcement and beams with inclined web reinforcement in only the tension direction were used. Only the data of the beam with 3-inch webs were included.

The product of a modulus of elasticity, such as that found from the cylinder tests, and the compressive strains found for the higher loads, gives compressive stresses which apparently are much higher than any which could possibly have been resisted by concrete of the strength used. This was taken as an indication that when cracks crossed the compression gauge line at a small angle with the direction of the gauge line or loosened one of the plugs as illustrated in gauge line 12, Figure 21, some slipping of the surfaces on each other took place which was measured as a shortening of the gauge length.

A study of the effect of varying the richness of the concrete indicated that for a given shearing stress the strains in compression were generally greater with lean mixtures than with rich mixtures.

[5] Tests of Heavily Reinforced Concrete Slab Beams; Effect of Direction of Reinforcement on Strength and Deformation.

Some of this variation may be seen in the variation of the slopes of the curves in Figure 46. However, the presence of diagonal tension cracks in the web would be expected to affect the distribution of compressive stresses in the web in an irregular manner. That it did so is indicated by inspection of Figure 47, in which the compressive strains in the webs of a considerable number of beams have been shown. The loads on all of the beams represented were such as to cause a computed shearing stress (uncorrected) of 600 lbs./in.[2] in the webs. Each point plotted represents the average of all the

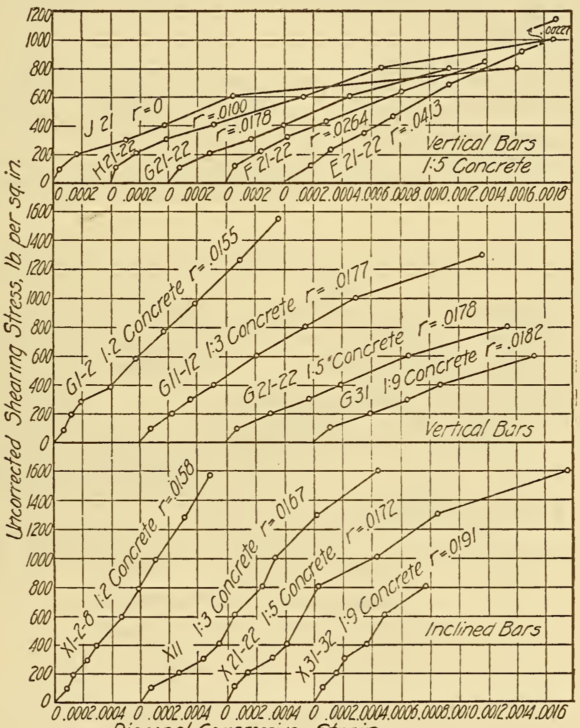

FIG. 46.—*Effect of variation in strength of concrete and in type of web reinforcement on diagonal compression in web*

diagonal compressions measured in the beam to which it refers. Usually the number of gauge lines was four. They were generally arranged as shown for gauge lines 9, 10, 11, and 12, beams 4J6 and 4YJ8, Figure 18. The compressive strengths of the concrete represented in Figure 47 are the averages for the respective mixes for all of the tests, and do not necessarily represent exactly the strengths for the beams referred to.

There were some indications that the compressive strains were somewhat smaller with beams which had large quantities of tension

reinforcement in the web than for those which had little or none. However, the effect of the amount of reinforcement was so slight and the behavior of the beams with varying amounts of web reinforcement was so irregular (see fig. 47) as regards the amount of compressive strain that the variation in the percentage of reinforcement was neglected and all values of the ratio of the shearing stress to the strain in compression were averaged. It was found from the average of 15 beams reinforced diagonally that the values of the ratio of shearing stress to compressive strain in the web was 1,294,000 lbs./in.^{2} In 28 beams reinforced vertically an average of 667,000 lbs./in.^{2} was found.

Putting e = strain, f_v = stress in diagonal compression in beams with vertical stirrups, and f_d = stress in diagonal compression in beams with diagonal stirrups, and writing these relations as equations,

$$\frac{v}{e} = 1{,}294{,}000 \text{ lbs./in.}^2 \text{ for diagonally reinforced beams} \tag{37}$$

and

$$\frac{v}{e} = 667{,}000 \text{ lbs./in.}^2 \text{ for vertically reinforced beams} \tag{38}$$

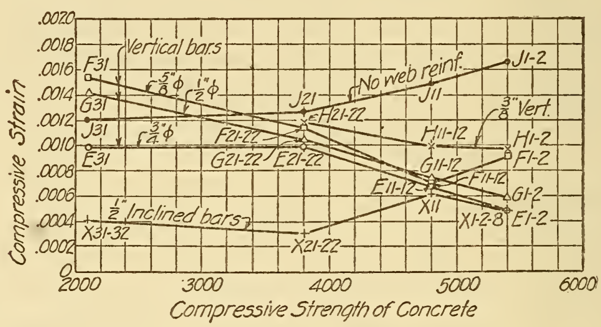

FIG. 47.—*Relation between compressive strain and compressive strength for uncorrected shearing stress of 600 lbs./in.^{2}*

Dividing both terms of both equations by the modulus of elasticity of the concrete (E_c) and solving for E_ce the equivalent of the concrete stresses f_v and f_d,

$$f_d = \frac{E_c}{1{,}294{,}000} v \text{ for beams with inclined web reinforcement} \tag{39}$$

$$f_v = \frac{E_c}{667{,}000} v \text{ for beams with vertical web reinforcement} \tag{40}$$

Dividing equation (40) by equation (39)

$$\frac{f_v}{f_d} = \frac{1{,}294{,}000}{667{,}000} = 1.94 \text{ when the shearing stress is the same for two types of beams} \tag{41}$$

For the beams which failed in diagonal compression equations (39) and (40) will generally give diagonal compressive stresses higher than any which it seems could have existed. This indicates, as has been stated before, that some deformation must have occurred, such as a slipping of the surface at cracks which was not a compressive strain. If it be assumed that the amount of slipping included in the measurements is proportional to the amount of strain in the gauge line, the relation shown in equation (41) may be taken to indicate that, on the whole, the diagonal compressive stress developed in vertically reinforced beams was approximately twice as great as that for diagonally reinforced beams. This is in conformity with the results of the analysis given in Section II, page 396, which indicates that the diagonal compression is twice as great for the beams reinforced vertically as for those reinforced diagonally at 45° with the axis of the beam. It is also in conformity with the observed fact

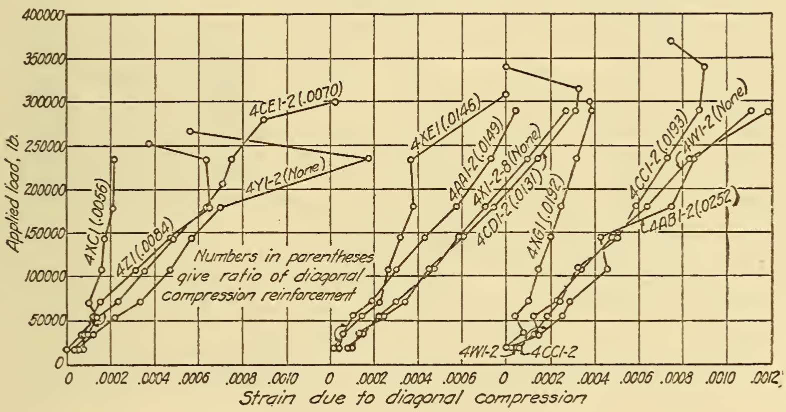

FIG. 48.—*Compressive strain in webs of typical beams with and without diagonal compression reinforcement*

that many of the beams with large quantities of vertical reinforcement failed wholly or partially by diagonal compression, while this was not true for any of the beams containing only inclined web reinforcement.

In Figure 48 the compressive strains in beams of a number of different types are shown for the purpose of comparison. In beams 4Y1, 2, 4X1, 2, 8, and 4W1, 2 there was no compression reinforcement in the web and the strains were measured on the concrete. All other beams had compression reinforcement in the webs and the strains were measured in the reinforcement. The beams having no compression reinforcement in the web showed about the same amount of compressive strain as did those with diagonal compression bars in the presence of diagonal tension or both diagonal tension and vertical web bars. The beams with expanded metal web reinforcement showed only about half as great diagonal compressive

strain as did the other beams. The strands made an angle of about 69° while the bars made an angle of 45° with the horizontal. It is probably the difference in angle that caused the difference in strain. (See also Sec. XVII, p. 444.)

There is no indication that the diagonal compression reinforcement served a useful purpose in any of the beams.

XIX. VARIATION IN SPACING OF STIRRUPS

In the beams deeper than 18 inches it was not feasible to make the spacing of the stirrups greater than about one-eighth of the depth from the compression surface to the tension reinforcement. With the spacing kept below this proportion of the depth it is believed that variation in spacing will have little effect on the behavior of the beam. Under existing regulations for design the spacing may amount

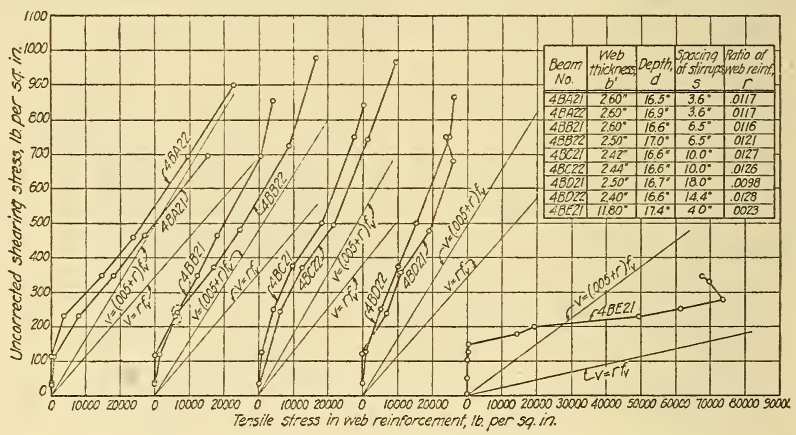

FIG. 49.—*Comparison of observed stresses in 18-inch beams with computed stresses*

to as much as 0.5 to 0.75 of the depth of the beam. In order to obtain information on the effect of variation in spacing, several beams of 18-inch depth were made in which the spacing varied from 3⅝ to 18 inches. The ratio of web reinforcement was nearly constant, and the size of the bars was increased as the spacing increased. The test results are given in Figures 49 and 50. The method given in Section VII, page 412, of correcting the shearing stresses for the effect of frame stiffness does not apply to these beams, and the shearing stresses shown are uncorrected. Figure 49 shows that for the beams having 3⅝-inch spacing of stirrups the tensile stresses in the web reinforcement conformed fairly well with the stresses given by formula (32), $v=(0.005+r)f_v$. As the spacing increased the tensile stresses at a given shearing stress decreased, and as a result the divergence from the stresses computed by formula (32)

increased. Beam 4BE21 had a web thickness of 12 inches and is discussed in Section XX, page 454. In Figure 50 the observed tensile stresses in the stirrups corresponding to shearing stresses of 200, 400, 600, and 800 lbs./in.[2] are shown, also the shearing stresses at maximum load. The tensile stress decreased quite regularly with an increase in the spacing of the stirrups. Apparently this indicates a lack in effectiveness of the stirrups with the larger spacing. This conclusion is borne out by the increase in the average crack width with the increase in spacing of stirrups. On the other hand, in spite of the ineffectiveness of the stirrups with large spacing the maximum shearing resistance fell off only slightly as the spacing increased, up to a spacing of 14.4 inches.

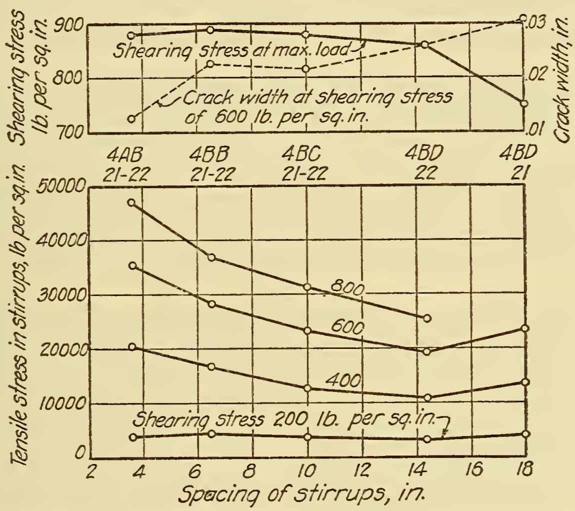

FIG. 50.—*Effect of variation in spacing of stirrups on tensile stresses developed*

On account of the loss of effectiveness of the stirrups with increase of spacing, it seems that vertical stirrups having a spacing greater than half the depth, d, of the beam should not be used. In cases of high working stresses in shear it will be desirable to use a still smaller spacing.

XX. VARIATION IN DEPTH OF BEAMS

The information furnished by these tests on the effect of variation of depth of beam on shearing strength is meager. In Table 4 are given the depths and spans, and the ratio of the distance between load and reaction (the half span) to the depth, d, of the beams.

The analysis (Sec. II, p. 390) assumes that the web of the beam is divided up into distinct diagonal elements, and therefore that the load and reaction are applied only to the elements which are attached to the horizontal members at the load points and reaction points,

respectively. Actually the beams tested were not so divided, and it is only at positions far enough away from the point of application of a load (or a reaction) for the stresses to become distributed somewhat uniformly over elements adjacent to each other, and for loads applied after the web was generally cracked into diagonal compression elements, that the analysis can be expected to give stresses comparable with those found in the tests. In some of the beams having a depth of 52 inches and a span of 16 feet stresses were observed in several stirrups along the length of the beam. These stresses have been plotted in Figures 51 and 54. The locations of gauge lines are shown in Figures 52 and 53. For all the beams having vertical stirrups shown in these figures the stress was greater in the stirrups which were intermediate between the load point and the support than they were in stirrups near the end of the center of the beam.

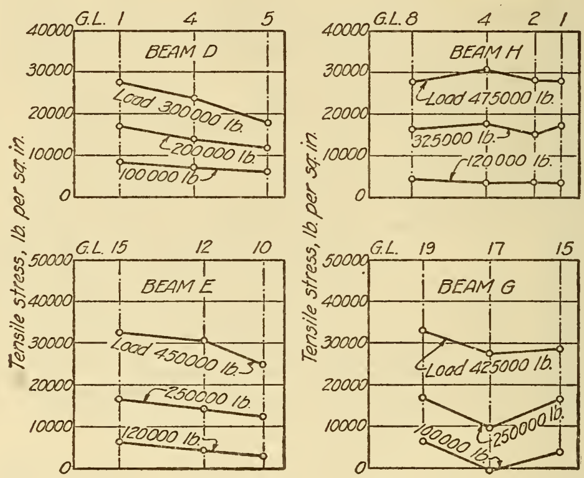

Fig. 51.—*Stresses in stirrups at varying positions along the length of beams 1D1, 1E1, 1G1, and 1H1*

Figure 54 also exhibits some tendency for the stresses in stirrups near the reaction to be greater near the bottom of the beam than at or above mid depth, and for those in stirrups near the load point to be greater at or above mid depth than near the bottom of the beam.

If the shortness of the beam relative to its depth had an effect on the distribution of stress it might be expected that the direct compression from the reaction would reduce the tensile stress in the end stirrups. If, however, this were the cause of the difference between stresses in the end stirrups and those in the center stirrups it might also be expected that the reduction of stress would be greatest near the bottom of the beam at the end, where the direct compressive stresses were most concentrated. The fact that instead the stresses

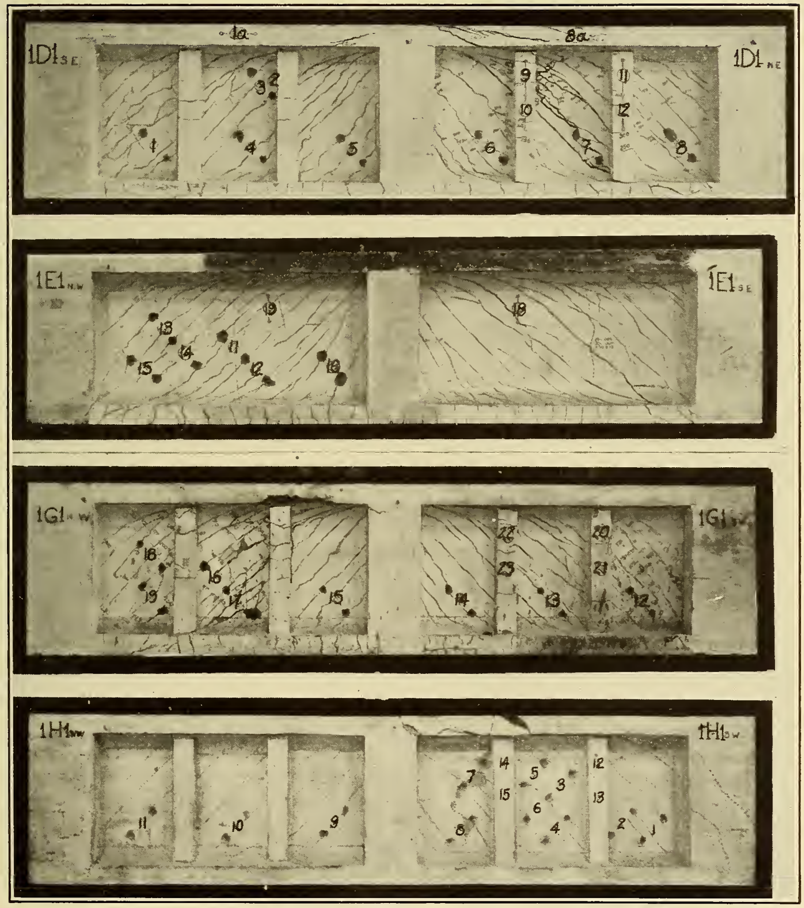

FIG. 52.—*Failure of 52-inch beams 1D1, 1E1, 1G1, and 1H1, showing location of gauge lines used in Figure 51*

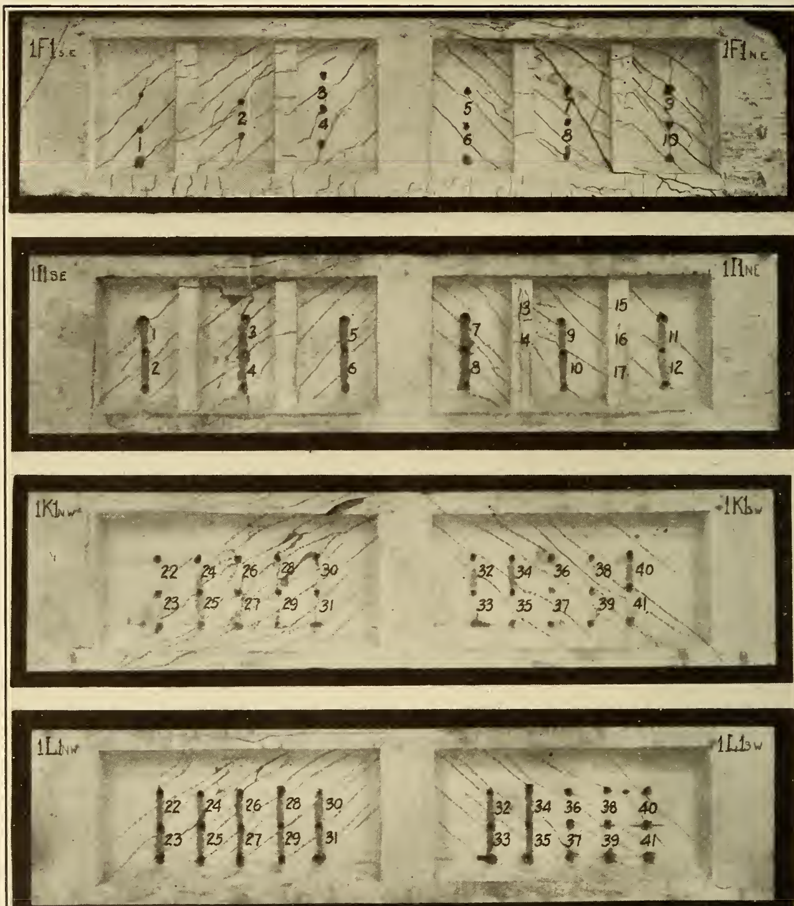

FIG. 53.—*Failure of 52-inch beams 1F1, 1I1, 1K1, and 1L1, showing location of gauge lines used in Figure 54*

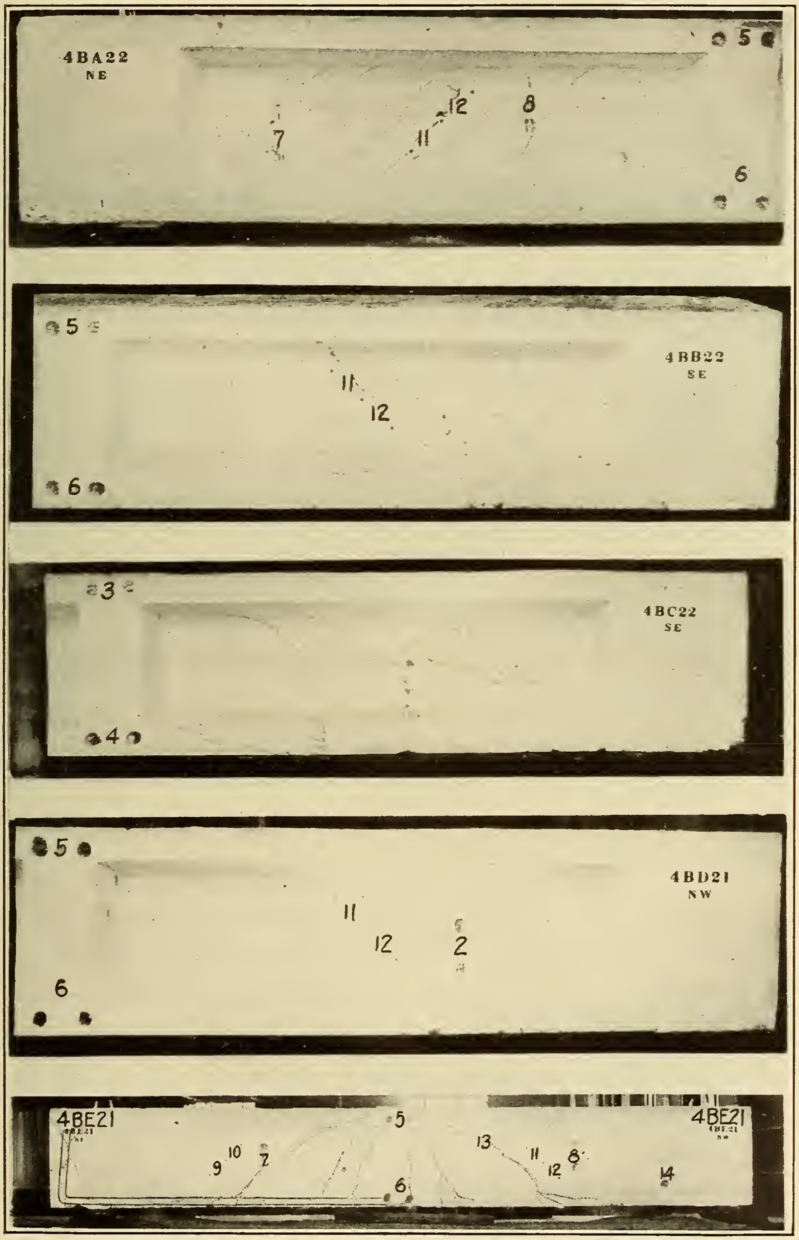

FIG. 55.—*Appearance of 18-inch beams after failure*

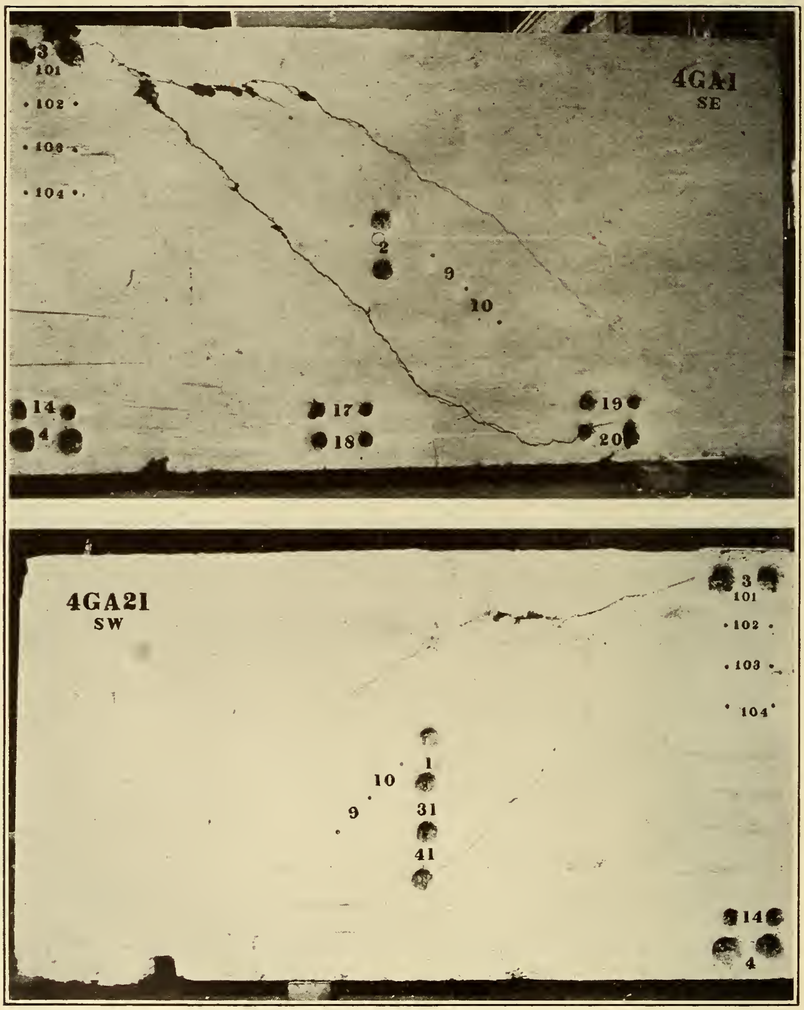

FIG. 56.—*Rectangular beams 4GA1 and 4GA21 after failure*

in the end stirrups were generally greatest at the bottom seems to indicate that the tensile stresses in the web were not reduced by the shortness of the beam below what they would have been in a longer beam subjected to the same shearing stress.

For the single beam 10 feet deep and of 20-foot span the shearing stress was higher at a given tensile stress in the web reinforcement than is shown by the formula derived from the data of the beams

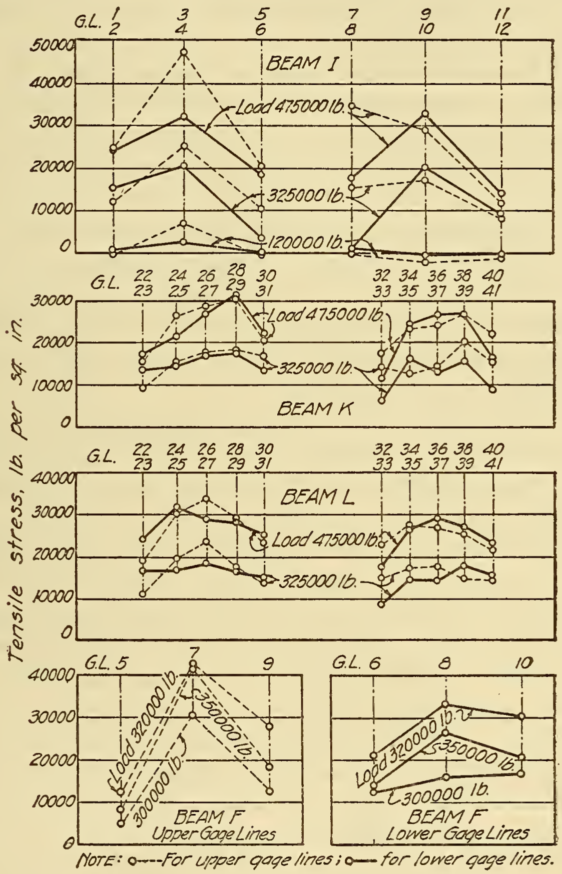

FIG. 54.—*Stresses in stirrups at various positions along the length of beams 1I1, 1K1, 1L1, and 1F1*

36 inches deep. (See equation (32).) This beam, however, failed by vertical shear when the tensile stress in the web reinforcement was only 26,000 lbs./in.², and there is no opportunity to make a comparison with other beams at higher tensile stresses. If the study of the relation between shearing stress and tensile stress in web reinforcement were confined to low loads the results for the other beams would be quite similar to those for beam 1X1. This is made clear by reference to Figures 38 and 39. It will be seen that for the

observed tensile stress of 20,000 lbs./in.^{2} most of the beams showed shearing stresses considerably greater than those given by the equation which for higher loads fitted the experimental points very well.

It has already been pointed out that the tensile stresses in the web reinforcement of beam 4BE21 were greater (see fig. 49) than those given by equation (32). This beam had a span of 9 feet 6 inches and a depth of 18 inches. It was rectangular in cross section and had only 0.23 per cent of web reinforcement. The stirrups were placed close to the outer faces to give access for taking strain-gauge readings. In each face they were 8 inches apart, but they were staggered so as to give a spacing of 4 inches. Therefore they were not well distributed in the beam, but comparison with other beams of the same depth indicates that the effectiveness of the stirrups was not disturbed by the uneven distribution. The cracks for this beam occurred near the center of the span (see fig. 55) instead of starting close to the supports, as is usual for beams in which the longitudinal bars were well anchored. The form of anchorage of the longitudinal bars was different for this beam from that of the majority of the beams, and the cracks at the left end, shown in Figure 55, give some indication of slipping of the bars.

Figure 49 shows that the rate of taking stress after cracks had formed was such that the slope of the curve is about the same as that of the graph of equation (27), $v = rf_v$, which applies to beams with vertical or 45° stirrups. The stresses were always less than those given by that equation, but greater than those given by equation (32), $v = (0.005 + r)\ f_v$, after a tensile stress of 28,000 lbs./in.^{2} had been reached. The behavior of this beam throws some question upon the general relations indicated by the other beams of the investigation. This question is the more perplexing because there is no other beam in the series exactly like it that serves as a check on the results shown, and the more important because for proportions between length, depth, and amount of reinforcement it is a type of beam likely to be much used in practice. It is important, however, that even for this beam, while the working stress in shear by formula (32) would be 116 lbs./in.^{2} ($v = (0.005 + 0.0023)\ 16{,}000$) when the tensile stress in the web reinforcement is taken as 16,000 lbs./in.^{2}, the factor of safety (ratio of ultimate shearing stress from Table 7 to the working stress) is $\frac{340}{116}$, or 2.93.

If the yield-point stress of the web reinforcement had been 40,000 instead of 60,000 lbs./in.^{2} the maximum load would probably have exceeded the load causing the stress of 40,000 lbs./in.^{2} by about the same amount as the actual maximum load exceeded the load causing the yield-point stress of 60,000 lbs./in.^{2}. On this basis the maximum

shearing stress would have been 300 lbs./in.^{2} and the factor of safety would have been 2.58.

To supply the lack of information for slender beams having a small amount of web reinforcement use has been made of data published in "Deutscher Ausschuss für Eisenbeton" No. 10, col. 8, Table 29, page 130. The beams referred to were T beams having a span of 118 inches (3 m) and were loaded at the one-third points of the span. The flanges were 19.7 inches (50 cm) wide and 3.93 inches (10 cm) thick, and the webs were 7.86 inches (20 cm) thick. The depth was 13.9 inches (35.3 cm) from the top of the beam to the center of the longitudinal reinforcement and 15.7 inches (40 cm) over all. The longitudinal reinforcement consisted in all cases of two bars 1.57 inches (4 cm) in diameter anchored by means of semicircular hooks at each end. Vertical stirrups of U shape 0.197, 0.276, and 0.394 inch (0.5, 0.7, and 1.0 cm), respectively, in diameter were used in the different beams. The spacings of the stirrups were 1.97, 3.94, 5.91, and 7.86 inches (5, 10, 15, and 20 cm), respectively. In any one beam all the stirrups were of the same size and the spacing was uniform. The age at test was 45 days. The concrete cubes showed a high degree of uniformity in strength. The average was 3,520 lbs./in.^{2} (248 kg/cm^2). This would be equivalent to a strength of about 2,570 lbs./in.^{2} for a cylinder having a height equal to twice its diameter. Strains in the stirrups were not measured, but "yielding of the stirrups" is one of the causes of failure assigned to all the beams. The yield point of the stirrups was 43,700 lbs./in.^{2} (3,065 kg/cm^2).

Assuming that failure occurred when the yield-point stress was reached, and that at other loads the stress in the stirrups was proportional to the loads, the shearing stresses corresponding to a tensile stress of 40,000 lbs./in.^{2} have been computed by taking $\frac{40000}{43500}$ times the shearing stress at maximum load as reported in Table 29, page 130, No. 10 "Deutscher Ausschuss für Eisenbeton." The shearing stresses have been plotted in Figure 38 for $f_s = 40{,}000$ lbs./in.^{2} These points fit in well with the other points of the diagram. It is recognized that the justification for some of the steps used in this comparison may be subject to question. For example, it is not likely that the beams failed as soon as the stress in the web reinforcement reached the yield point. The amount of correction which was applied was not sufficient, however, to destroy the value of the comparison made.

Though more data on this subject are to be desired, it appears likely that the deductions from the tests of the deeper beams may safely be applied to more slender beams.

XXI. ANCHORAGE OF STIRRUPS

At the time that the tests of series 1 were started doubt existed as to the possibility of developing very great shearing resistance in concrete webs by the use of large percentages of web reinforcement, on account of the difficulty in securing proper anchorage of stirrups. In the first tests, series 1, the stirrups in most of the beams were anchored as shown in Figure 9 (*H*). In certain beams the stirrups were anchored by welding to the longitudinal steel at top and bottom of the beam, as shown in Figure 9 (*I*). The result of these tests showed conclusively that the anchorage without welding was sufficient to prevent slipping of the stirrups under the conditions in those beams. In the early beams of series 4 the stirrup anchorage was effected in a similar manner, as shown in Figure 9(*A*). In later beams types of anchorage were used that involved less complicated bending of the ends of the stirrups, as indicated in Figure 9 (*B*) and Figure 9 (*C*). There was no indication that the type of anchorage shown in Figure 9 (*C*) was any less effective than the other types.

Beams 36 inches deep and reinforced by web bars 1 inch in diameter appear to have failed in all cases by slipping at the upper ends of the bars. All these beams had diagonal stirrups anchored as shown in Figure 9 (*A*). Beams with three-fourths inch web bars (either vertical or diagonal) and with 3-inch webs also appear to have failed by slipping. The stirrup anchorage was as shown in Figure 9 (*A*) or Figure 9 (*B*). However, beams with three-fourths inch vertical web bars and with 6-inch and 8.5-inch webs failed by diagonal tension upon reaching the yield point of the stirrups, and not by slipping. In these beams with thicker webs the stirrup anchorage was as shown in Figure 9 (*C*), and it can not be positively stated whether it was the different type of anchorage or the greater web thickness which prevented the slipping. In no case did beams 36 inches deep with stirrups five-eighths inch or less in diameter, or beams 52 inches deep with stirrups three-fourths inch or less in diameter, fail on account of slipping of the stirrups.

It will be seen in Figures 39 and 41 that beam 4AK2, which had 1-inch inclined bars in a web 3 inches thick, showed stresses so great as to throw the data for that beam out of line with the data of the other beams. It is likely that the slipping of the bars which were anchored in the top of the beam near the support threw more than the usual amount of tensile stress into the other web bars at the place where the strain was measured.

XXII. ANCHORAGE OF LONGITUDINAL BARS AND TENSILE STRESSES DEVELOPED

The arrangement of the anchorage is shown in Figure 7. In all the beams the longitudinal steel in the tension side of the beam was anchored, usually by means of semicircular hooks. Generally where only one layer of 1¼-inch bars was used the hook was 15 inches in mean diameter. In series 1 two layers of 1¼-inch bars were used; those in the lower layer were bent to a mean diameter of 13 inches and those in the upper layer to a mean diameter of 8.5 inches. In series 4 the bars of the lower layer were bent to a 15-inch diameter and those in the upper layer to either a 15-inch or a 10-inch diameter.

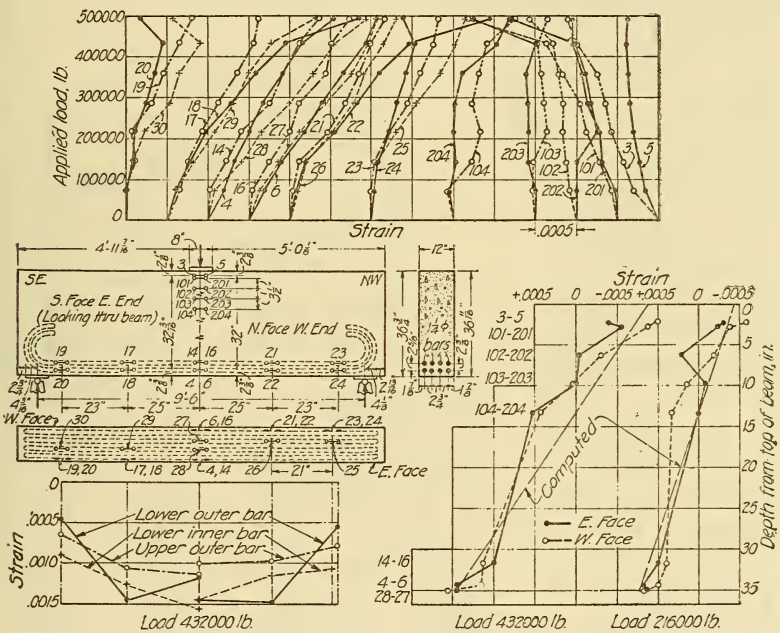

FIG. 57.—*Observed strains in 28 longitudinal gauge lines in beam 4GA1*

The length of bar beyond the end of the curve varied from 2 to 4 inches. In beam tests it has sometimes been difficult to distinguish web failures caused by slipping of the longitudinal bars from true diagonal tension failures. It was to preclude the possibility of such confusion that the longitudinal tension bars of all beams were anchored.

In order to obtain data on the amount of stress developed by bond and the amount developed by the anchorage, strain readings were taken in the straight portions of the reinforcing bars of the rectangular beams 4GA1 and 4GA21 at the center and at the one-quarter points of the span and as close as possible to the support. The results of these observations, together with other data of the tests, are shown in Figures 57 and 58. These results show that bars in different

positions in the beam behaved differently, and that the two beams were consistent with each other in this respect. For a load of 432,000 pounds the outer tension bars of the lower layer showed smaller stresses close to the support and greater stresses at the one-quarter

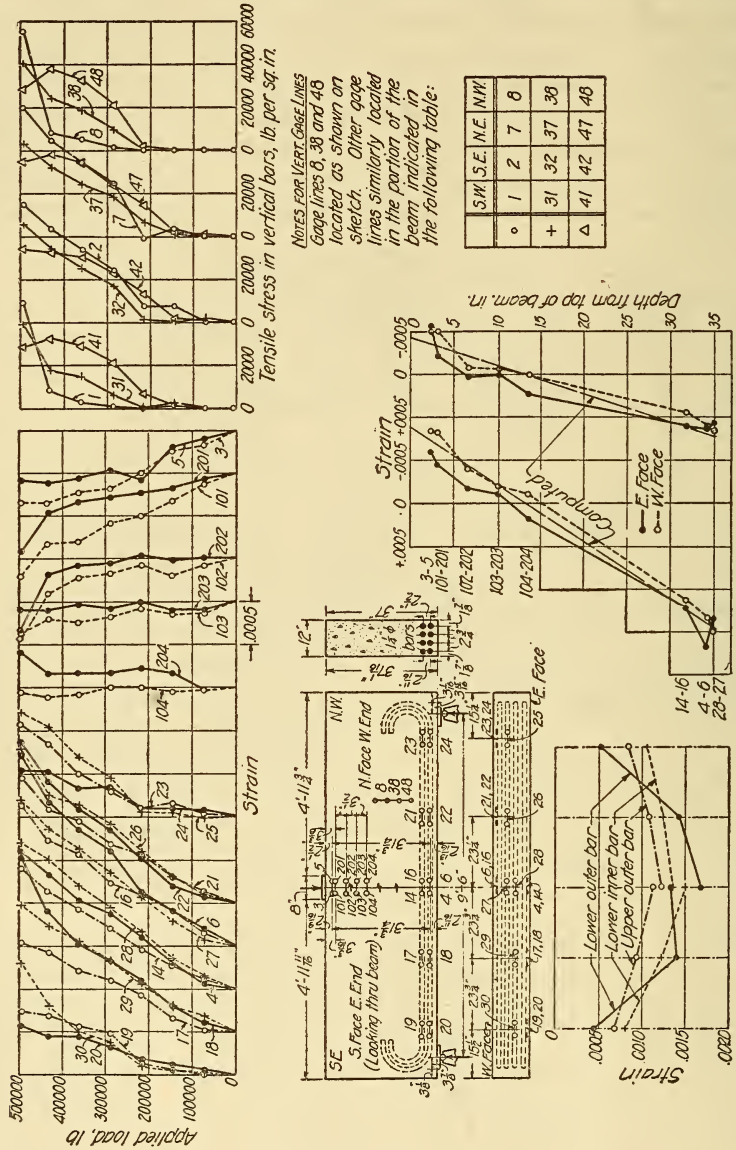

Fig. 58.—*Observed strain in longitudinal gauge lines in beam 4GA21*

point than did any other bar on which observations were taken. The outer tension bars of the upper layer showed stresses of intermediate value near the support and smaller stresses at the one-quarter point and at the center of the span than did any other bar

observed. At the center of the span the average stress for the upper and lower outer bars was, in general, about equal to the stress observed in a bar of the bottom layer lying near to the longitudinal center line of the beam.

In most cases cracks formed on the sides of the beams at the ends of the bars, as illustrated in Figure 18. These cracks followed quite closely the contour of the hooks by means of which the bars were anchored. Generally there was only about 1¼ inches of concrete between the surface of the bar and the side surface of the beam, and it was believed that the cracks represented a tendency of the bars to loosen and pull out sidewise as the anchorage came into action. This belief is strengthened by the fact that gauge lines 20 and 24 in beam 4GA1 show, in Figure 57, that the stress in the outside bars fell off near the support when the load was increased from 432,000 to 496,000 pounds. The same falling off of stress is seen for gauge line 30, which was on an interior bar. The latter is probably to be explained by the fact that, as stated in the notes of the test, there was a wide longitudinal crack on the under side of the beam extending from gauge line 29 to gauge line 30. This crack probably extended far enough to loosen the anchorage of the interior bar.

All of these indications point toward a loosening of the anchorage of the bars at high loads by cracking of the concrete on the sides of the beam around the hooks of the bars and by splitting vertically through the center of the beam. Some indications of the same nature, but less marked, are present in beam 4GA21. (See fig. 58.) The fact that in both beams 4GA1 and 4GA21 the highest stresses of all were in the outside bars (see gauge lines 18 and 22 for beam 4GA1, fig. 57, and gauge lines 6 and 18 for beam 4GA21, fig. 58) seems to indicate that with the embedment of 1¼ inches the outer bars were as well anchored as the interior bars.

On the other hand, in beam 4E9 the slipping of the outer bars was sufficient to split off a portion of the concrete at the end of the beam. Figure 21 shows this beam as it appeared after failure. A careful examination of the beam gave evidence of the slipping of both the upper and lower bars of the bottom flange nearest the side of the beam. The concrete within the hook of the lowest bar and outside the hook of the upper bar was somewhat crushed. The cracks in the pilaster of this beam were not as large as those found in some other beams. The second bar from the surface had not slipped.

The average observed tensile stress at the center of the span for all bars measured in both beams was very near 40,000 lbs./in.^{2}, and that close to the support was 23,000 lbs./in.^{2}; that is, 58 per cent as great. Assuming perfect bond between the steel and the concrete, only about 14 per cent of the stress at the center of the span should be expected at the gauge lines near the support. The computed

bond stress, $\frac{V}{\Sigma ojd}$, at this load was 240 lbs./in.[2] The one thing which stands out clearly in these figures is that the hooks were called into play and that at a bond stress about three times that ordinarily used for working loads the hooks apparently were furnishing about half the anchorage of the bars. These observations emphasize the importance of designing with low bond stresses or of anchoring the ends of the bars.

The strains observed at different depths on opposite sides of beam 4GA1 are plotted in Figure 57 at the right for loads of 216,000 and 432,000 pounds. The straight lines which indicate the computed strains are shown also. The agreement between the observed and the computed values is fair for the top and the bottom of the beam, but the observed strains do not indicate a "straight line" stress distribution over the depth of the beams.

For comparison with the observed stresses the computed tensile stresses in the longitudinal reinforcement have been shown by means of dotted lines (marked "Computed stress") in Figure 76 for the beams listed in Table 5. Certain other significant features are shown in the table.

TABLE 5.—*Beams used for comparison of observed with computed tensile stresses in longitudinal reinforcement*

Beam number	Reference number Table 3	Number of bars		Flange width	Web thickness
		Tension	Compression		
				Inches	*Inches*
4YG1	35	5	1	12	3
4YF1	22	6	1	12	3
4YE1	6	7	1	12	3
4E3	11	8	8	12	4
4G3	38	8	8	12	4
4E6,7	12	10	10	15	6
4G6	40	10	10	15	6
4E9	13	12	12	17.5	8.5
4G8	41	12	12	17.5	8.5

It will be seen that the observed stresses were generally less than the computed stresses, though in some instances at the higher loads the observed stresses reached or exceeded the computed stresses. Since the gauge length used was only 4 inches and the bars were large (1¼ inches), the maximum stress at a crack can not have been much greater than that indicated by the average strain within the gauge length.

The measurements of tension were generally taken on only the outside bars of the lower layer, while the computed stresses represented the average for all the bars. The comparison of observed with computed stress should be made with the fact borne in mind that there was a considerable variation between stresses in upper and lower layers of tension bars and between exterior and interior bars.

XXIII. CRACK WIDTHS

In Section V, page 408, on testing it is stated that the widths of the cracks in the webs were measured after each increment of load. Numerous checks by independent observers on these measured widths indicate that the widths so determined were accurate to the nearest 0.002 inch. In order to make use of the data in the study of the results a graph was plotted for each beam in which the ordinates were the average widths of the five largest cracks and the abscissas were the uncorrected shearing stresses. Typical graphs of this kind are given in Figure 59. The regularity of the plotted curves furnishes an additional check of the reliability of the data. Graphs of crack widths against total loads are given for all the beams in Figure 78.

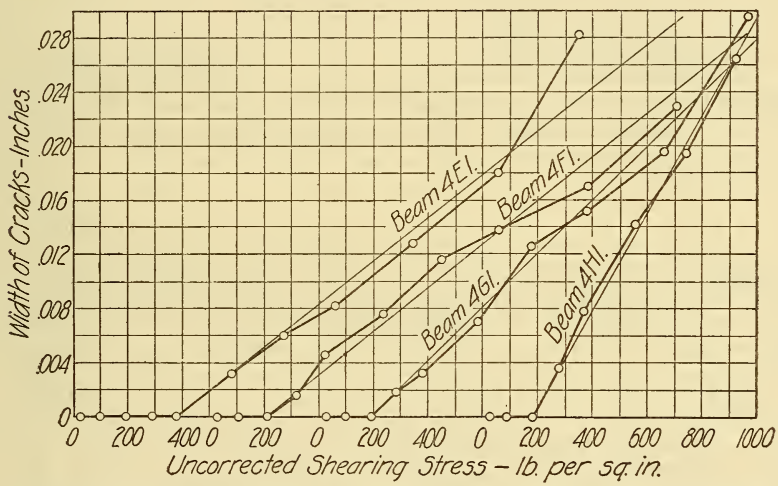

Fig. 59.—*Crack widths at various uncorrected shearing stresses for specimen beams*

A characteristic common to practically all the test beams is that up to a shearing stress of 100 to 300 lbs./in.^{2} no cracks were present, and that after the cracking began the amount of widening was approximately proportional to the rate of adding to the load. Straight lines fitted to the portion of the crack-width curves beyond the point at which the first crack occurred intersect the axis of zero crack widths at a point which corresponds fairly closely to the shearing stress at which the first crack occurred. That intercept has been used as a measure of the resistance of the beam to the formation of cracks due to diagonal tensile stress. The slope of the crack-width curve has been used as a measure of the rate of widening of the cracks. In Figures 60 to 64 the shearing stress, i (uncorrected), at which the first crack occurred and the rate of widening of the

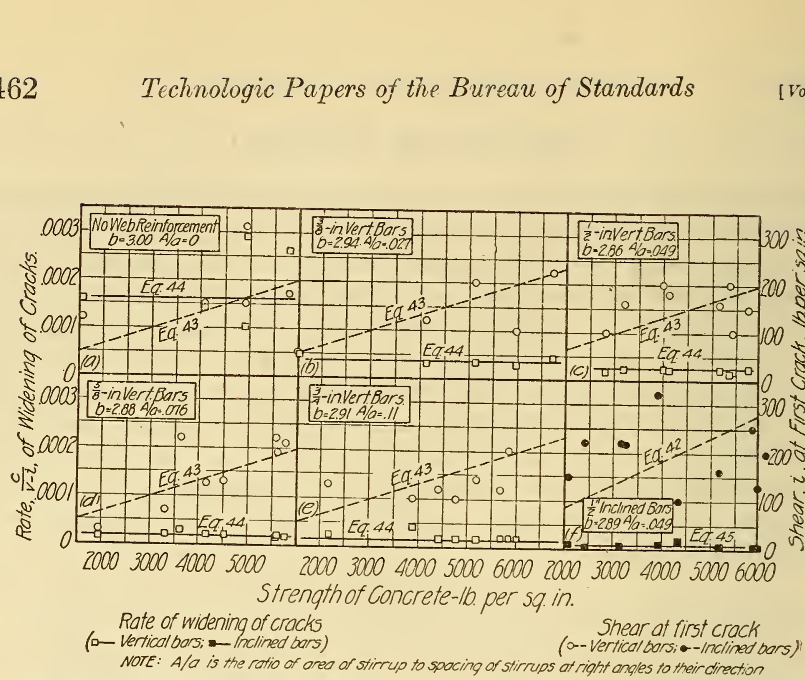

FIG. 60.—*Relation of rate of widening of cracks and uncorrected shear at first crack to strength of concrete for beams with 3-inch webs*

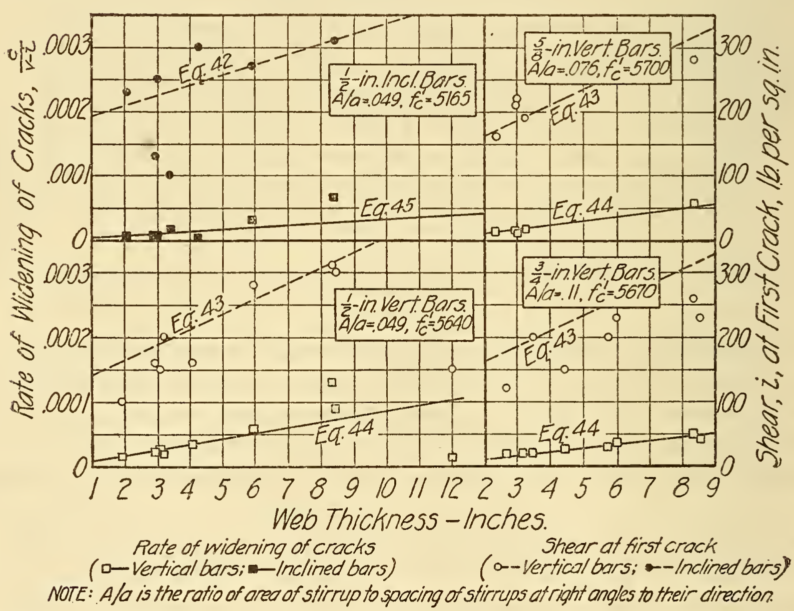

FIG. 61.—*Relation of rate of widening of cracks and uncorrected shear at first crack to thickness of web for beams with 1 : 2 concrete*

cracks have been plotted as ordinates, and various properties of the beams have been shown as abscissas. These figures permit the study of the effect of variation in spacing of stirrups, amount of web reinforcement, web thickness, and strength of concrete on the shearing stress at which the first cracks occur, and on the rate of widening of the cracks. In these diagrams and in the following paragraphs the shearing stress at which the first crack occurred is represented by the symbol i, and the rate of widening of the cracks by the expression $\frac{c}{v-i}$. In the latter expression c is the width of the crack in inches and v is the shearing stress on the beam.

A study of Figures 60 to 64 indicates that the shear i, at which the first crack formed, was practically independent of the spacing of the vertical stirrups and of the amount of either vertical or horizontal web reinforcement. The value of i was distinctly affected by variation in the web thickness and in the strength of the concrete. It was also higher for beams with inclined stirrups than for beams with vertical stirrups, and seemed to be affected somewhat by the amount of inclined web reinforcement. Since vertical stirrups can be of little or no value until after the concrete of the web has cracked, it is not surprising that they showed no effect on the shearing stress at which the first crack occurred.

The formation of the first crack must be determined largely by the tensile strength of the concrete web, and it is seen that the web thickness and the compressive strength of the concrete, both of which affected the shearing stress at which the first crack occurred, would be expected to affect the tensile strength of the concrete web. It will be noted that the properties of the beam which had an effect on the shearing stress at which the first cracks occurred were generally those properties which had an effect on the tensile strength of the concrete web.

It will be seen from these diagrams, Figures 60 to 64, that the rate of widening of the cracks, $\frac{c}{v-i}$, was independent of the amount of horizontal web reinforcement and of the compressive strength of the concrete. It was slightly dependent upon the amount and spacing of the vertical reinforcement and upon the thickness of the webs. For beams with small amounts of web reinforcement the rate of widening of the cracks was markedly dependent upon the amount of web reinforcement. With large amounts of web reinforcement less of this effect was apparent. The rate of widening of the cracks was much less for beams with inclined stirrups than for beams with vertical stirrups.

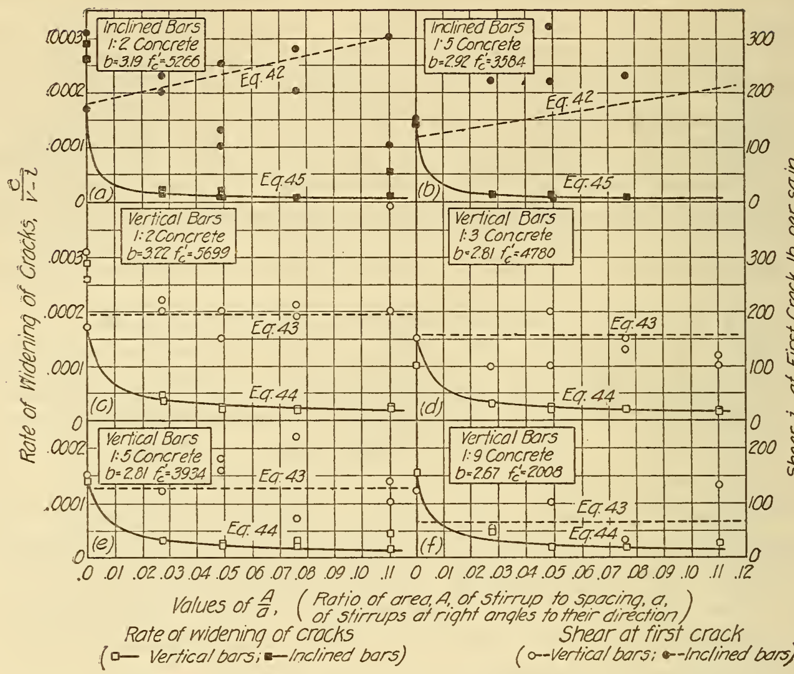

FIG. 62.—*Relation of rate of widening of cracks and uncorrected shear at first crack to quantity of vertical and inclined web reinforcement for beams with 3-inch webs*

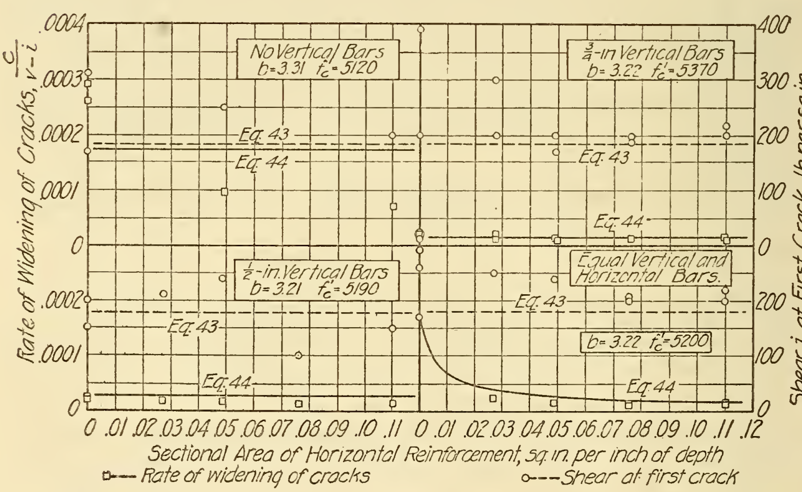

FIG. 63.—*Relation of rate of widening of cracks and uncorrected shear at first crack to quantity of horizontal web reinforcement for beams with 3-inch webs and 1 : 2 concrete*

The foregoing statements are qualitative. For some of the relations rather definite quantitative values may be stated. The equation

$$i=\left(\frac{5+b'}{240}+\frac{16-b'}{60}\cdot\frac{A_d}{a_d}\right)f'_c \tag{42}$$

represents fairly well the relation between the shearing stress i at which the first crack occurred, the web thickness b', the compressive strength of the concrete f'_c, and the ratio $\frac{A_d}{a_d}$ of inclined web reinforcement. For a beam with vertical stirrups only, or with no web reinforcement at all, this equation becomes

$$i=\left(\frac{5+b'}{240}\right)f'_c \tag{43}$$

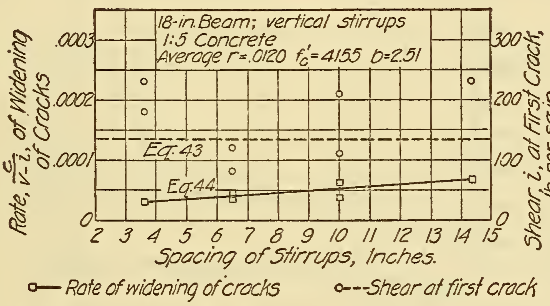

FIG. 64.—*Relation of rate of widening of cracks and uncorrected shear at first crack to spacing of vertical stirrups for 18-inch beams of 1 : 5 concrete; ratio of web reinforcement 0.0120*

These equations repeat quantitatively the previous statements that the shearing stress at which the first crack occurred was independent of the amount of vertical web reinforcement, and that it was slightly dependent upon the amount of inclined reinforcement.

The lines fitted to the circles in Figures 60 and 61 are graphs of equations (42) and (43). The graphs fit the points for beams with vertical stirrups fairly well throughout, but for those with inclined stirrups the agreement is not so good. In Figure 60 (f) a horizontal line would fit the points nearly as well as the inclined line, but the fact that there should be a slope to the curve seems to be fairly well established by the other curves.

The fact that the rate of crack widening was independent of the compressive strength of the concrete makes available for studying the effect of variation in quantity of reinforcement on the rate of widening of cracks a much larger number of data than otherwise would be available. All values of the slope $\frac{c}{v-i}$ for a given value of the same ratio of web reinforcement may be averaged regardless of

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the strength of the concrete. This is indicated by the fact that in Figure 60 the lines which fit the course of points shown as squares are horizontal. In Figure 65 the average values of $\frac{c}{v-i}$ for the beams with 3-inch webs have been plotted as ordinates and the corresponding values of $\frac{A}{a}$ as abscissas for both vertical and inclined web reinforcement. Not so many values are available for beams with inclined as for beams with vertical reinforcement. Equations have been fitted to the points thus found. In these equations it has been recognized that the rate of widening of the cracks varied with the web thickness as well as with the amount of web reinforcement. The term $\frac{b'}{3}$ in the expression takes account of this fact. It is possible

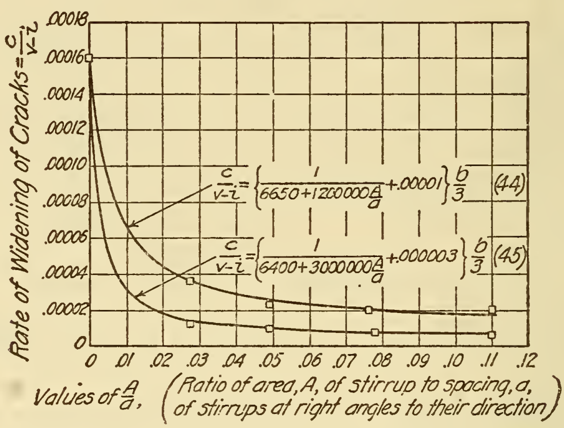

Fig. 65.—*Rate of widening of cracks for beams with 3-inch webs*

that a term should be included which will take account of the spacing of the stirrups, but the data of Figure 65 were not sufficient to warrant doing so. The equations showing the rate of widening of cracks are

$$\frac{c}{v-i}=\left(\frac{1}{6650+1200000\frac{A}{a}}+0.00001\right)\frac{b'}{3} \tag{44}$$

for vertical stirrups and

$$\frac{c}{v-i}=\left(\frac{1}{6400+3000000\frac{A}{a}}+0.000003\right)\frac{b'}{3} \tag{45}$$

for inclined stirrups.

The graphs of these equations have been drawn in Figures 60 to 64. From these diagrams it may be seen that the equations repre-

sent fairly well the effect of the different variables involved in them upon the resistance of the beams to the formation of diagonal cracks, and upon the rate of widening of the cracks. The constants used in the equations for the various cases are those which appear in the legend for the different diagrams.

It is hardly to be expected that the equations for shearing stress at first crack and for rate of widening of cracks will apply without modification to beams of widely different shapes and proportions.

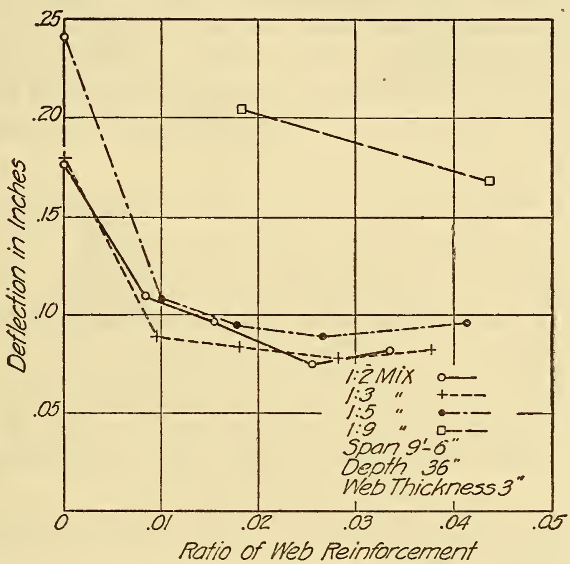

FIG. 66.—*Deflections for beams with vertical web reinforcement; uncorrected shearing stress 800 lbs./in.²*

The principal value of expressing these relations by empirical formulas is probably that it shows that there was less accident and more law in the formation of cracks than may be generally recognized.

XXIV. DEFLECTIONS

Figure 66 is a diagram which shows the deflection of beams having (1) only vertical reinforcement in the web, (2) the same web thickness, (3) web concrete of varying strength, and (4) varying percentages of web reinforcement. This diagram is plotted for loads which gave an uncorrected shearing stress of 800 lbs./in.² for all beams. It will be seen from this diagram that there was practically no difference in the deflections due to variations in richness of concrete except for the 1 : 9 mix. This is probably accounted for by the fact that beams having a 1 : 9 mix were either at or very near the point of failure by compression in the web at the load which gave a shearing stress of 800 lbs./in.²

Figure 67 shows deflections as ordinates and strengths of concrete as abscissas for beams with concretes of varying strengths and with vertical web reinforcement. Within the range of strengths used there was little increase in deflection with decrease in strength of concrete. This confirms the conclusion regarding Figure 66.

In Figure 68 are plotted for shearing stresses (uncorrected) of 800 lbs./in.^{2} the deflections for beams of the same web thickness having diagonal web reinforcement only, but with varying strengths of web concrete and varying percentages of web reinforcement. The deflections varied irregularly, and only slightly with the variation in richness of the concrete. The decrease in deflection with increase of amount of web reinforcement was most pronounced for the small ratios of reinforcement, but it was quite regular throughout the range of the tests.

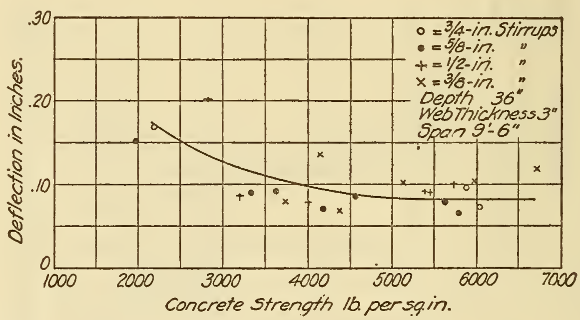

FIG. 67.—*Deflections for beams with vertical web reinforcement and varying strengths of concrete; uncorrected shearing stress 800 lbs./in.^{2}*

Figure 69 shows a comparison of the deflections for shearing stresses of 800 lbs./in.^{2} in beams having diagonal tension reinforcement only with those for beams having both diagonal tension and diagonal compression reinforcement. The fact that the curves cross each other and that at all points they are close together seems to justify the belief that, in general, there would be little difference in the deflection for these two types of beams.

A comparison of Figures 66 and 68 indicates that as a rule the deflections for beams with vertical reinforcement were larger than those for beams with inclined reinforcement at the higher percentages of web reinforcement, but that as the amount of web reinforcement became smaller the deflections approached each other until for a beam with no web reinforcement the two necessarily coincide.

Variation in the thickness of web showed a distinct effect on the amount of deflection. In Figure 70 the deflections are shown for shearing stresses of 800 lbs./in.^{2} in beams having a 1 : 2 mix, vertical web reinforcement, and varying web thicknesses. From this diagram

it will be noted that for the same shearing stress beams with thick webs showed considerably higher deflections than beams with thin webs. It is possible that the excess deflection for beams with thick

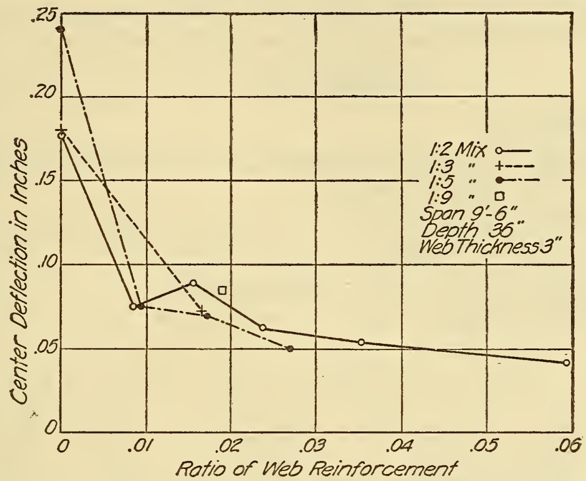

FIG. 68.—*Deflections for beams with inclined web reinforcement; uncorrected shearing stress 800 lbs./in.*^{2}

webs may be due to the higher bond stresses which must have been present in the web reinforcement for a given shearing stress. This higher bond stress is due to the use of large rods to obtain the same

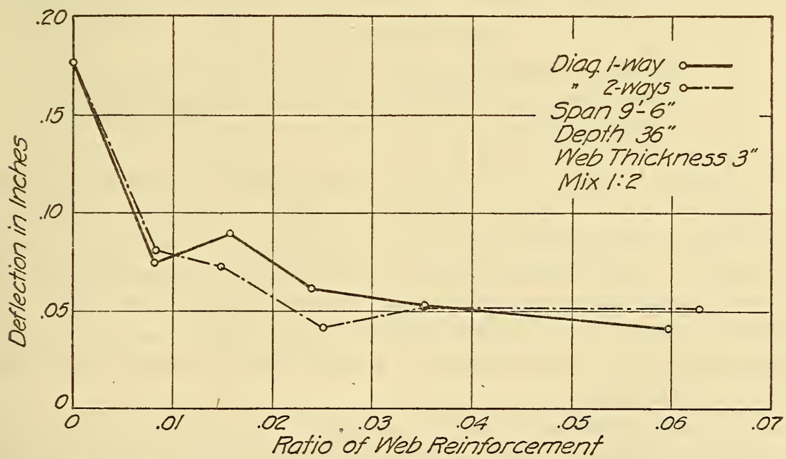

FIG. 69.—*Deflections for beams with diagonal tension and diagonal compression reinforcement; uncorrected shearing stress 800 lbs./in.*^{2}

percentage of reinforcement in the thick webs as was obtained in the thinner webs by the use of small rods. The phenomena of deflection and those of crack width are similar in that a thickening of the web caused an increase in both the crack width and the deflection

for a given shear, and it is likely that the presence of higher bond stresses for the thicker web is responsible for both phenomena.

In Figure 71 the measured deflections for several beams are plotted, and in the same figure are shown the deflections for the same beams computed from the measured strains by the formula

$$\Delta = \frac{kl^2}{d}(e_c + e_s) \qquad (46)$$

in which

Δ = deflection,
k = 0.0833, a constant depending upon method of loading,
l = 114, the span in inches,
d = vertical distance between measured strains at top and at bottom of beam,
e_s = strain in lower point of measurement,
e_c = strain at upper point of measurement.

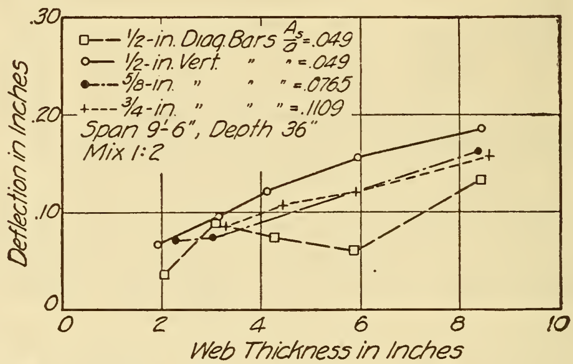

FIG. 70.—*Deflections for beams with vertical reinforcement and varying web thickness; uncorrected shearing stress 800 lbs./in.²*

For beams having low shearing stresses and few diagonal cracks the agreement between the measured deflections and the deflections calculated by this formula has been found in previous studies to be very good.[6] For the tests under consideration the agreement was not good, and it was worse for high loads than for low loads. The shearing deflections and the vertical opening of the cracks will probably account for the difference between the observed and the computed deflections.

In the testing of beam 4H2 vertical deflections were measured on the upper and lower flanges, and horizontal deflections were measured at one end of the beam. The vertical deflections were measured (by means of a mirror and scale attached to the beam at each point indicated by a small rectangle in the sketch of the beam in fig. 72) from a thread stretched between pins attached at the ends of the

[6] G. A. Maney, "Relation between deformation and deflection in reinforced concrete beams." Proc. Am. Soc. Test. Mats., 14, p. 310; 1914.

beam. The horizontal deflections were measured from a weighted thread hung from the pin at the upper right-hand corner. Assuming that the movements were symmetrical about the center of the span, the vertical deflections were as shown in Figure 72 below the elevation of the beam. The total horizontal deflections are shown in the small rectangle to the right of the elevation of the beam. It will be seen that the end pilaster of the beam tilted as the beam deflected,

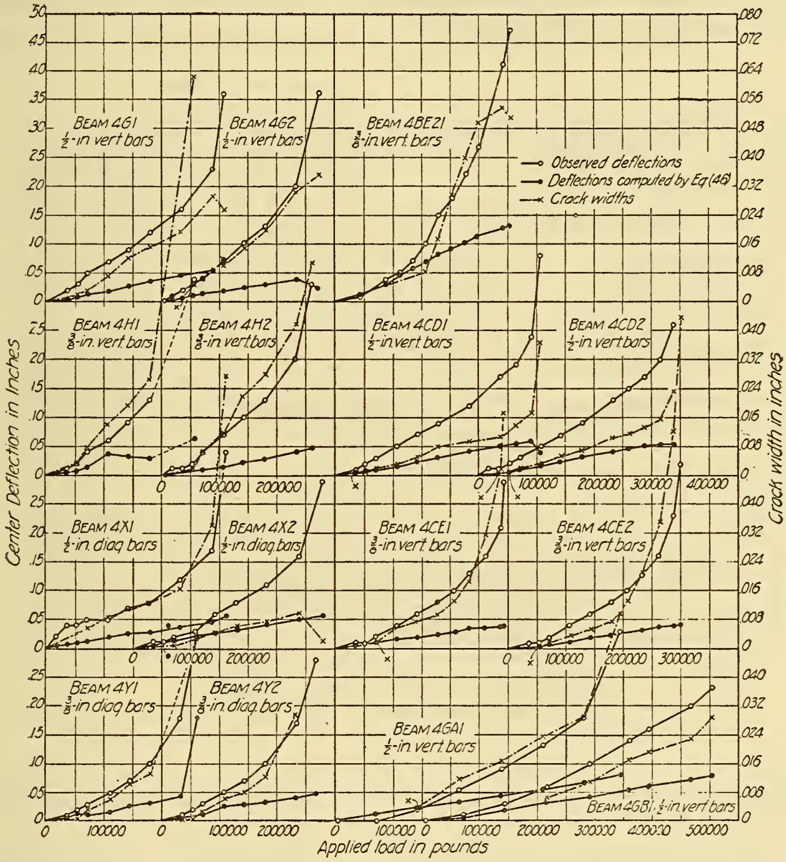

FIG. 71.—*Measured deflections compared with deflections computed from strains*

and was distorted only slightly. Neglecting this distortion and assuming that the top moved toward the center as much as the bottom moved away from the center, the deflected position of the end of the beam under maximum load is shown as a dotted line in the elevation of the beam. The upper and lower surfaces of the deflected beam are formed by the maximum-load deflection curves taken from the diagram below.

A curve has been drawn through zero deflection at the reactions and through the point which represents the center deflection computed by equation (46) for the strains, e_c and e_s, observed at the maximum load of 261,500 pounds.

Outside the center of the support the observed slope of the beam agrees fairly well with the slope of the curve showing the computed center deflection. It may be assumed that the excess of the observed over the computed center deflection represents the shearing deflection plus the deflection caused by the opening of cracks in the web.

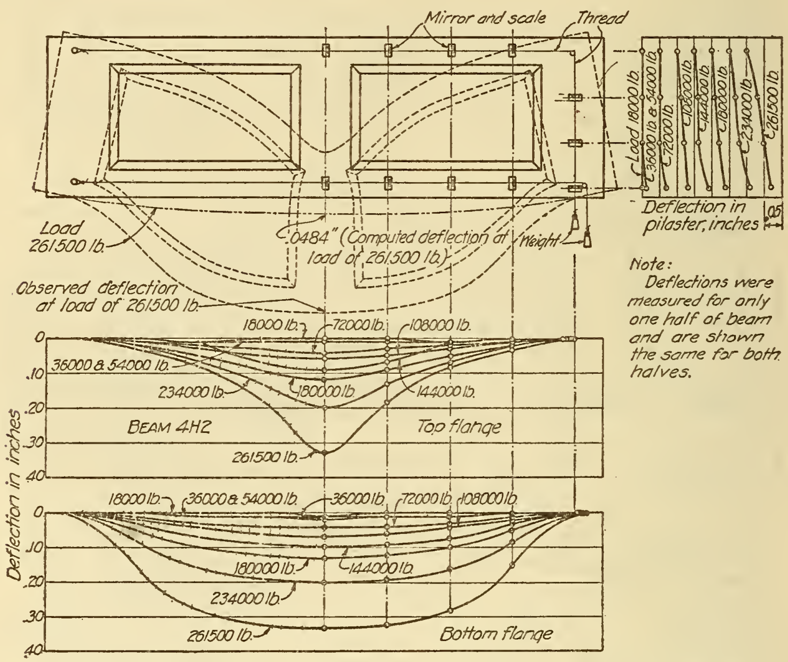

Fig. 72.—*Deflections at various points on span for beam 4H2*

Deflections were measured for only one-half of the beam and are shown the same for both halves. The upper and lower surfaces of the deflected beam are formed by the maximum-load deflection curves shown in the diagram above.

The upper and lower flanges did not remain parallel to each other in the deflected position of the beam. The flanges were farther apart at intermediate portions of the beam than at the center of the span or at the support. This is consistent with the fact that there was a tendency for the stresses to be greater in the vertical stirrups at intermediate points than in those near the center or the ends of the beam. See Figures 51 and 54, and Section XX, pages 452 and 453.

In Figure 73 diagrams of the deflections in half spans of a number of beams are shown. The spreading apart of the top and bottom flanges between load point and support is seen to be a phenomenon common to most of the beams. For the beams having one-half

inch vertical stirrups and no concrete web (4G5) the stirrups apparently held the flanges from spreading apart. The differences in amount of deflection and shape of deflection curves for the beams shown in this figure help to visualize the function of the concrete web and the web reinforcement in the behavior of the beams. The conditions which caused the deflection of the upper flange of beam 4AG21 to be greater than that of the lower flange are not known. This could be caused by horizontal cracks in the end pilasters which permitted the extreme ends of the upper flange to rise, but there is no evidence that such cracks were present.

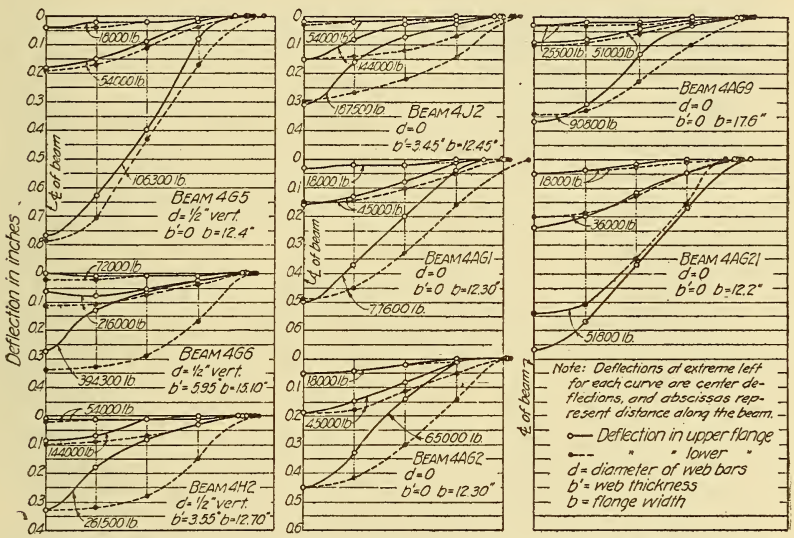

FIG. 73.—*Deflections at various points on span for beams 4G5, 4G6, 4H2, 4J2, 4AG1, 4AG2, 4AG9, and 4AG21*

Deflections at extreme left are center deflections, and abscissas represent distance along the beam

Comparing the center deflections shown in Figure 79, it is found that at a load of 50,000 pounds the deflection of a beam with one-half-inch diagonal web bars and no concrete web (4X5) was only slightly greater than that of the beams with one-half inch diagonal bars and a 3-inch web (4X1, 2 and 8) or of the beams with 3-inch web and no web reinforcement (4J1 and 2). For beams with no concrete in the web the deflection of those with one-half-inch diagonal web bars was much less than that of beams with vertical web bars or with no web reinforcement at all. These phenomena indicate clearly that the inclined bars became effective much earlier in the test than did the vertical bars, and indicate a superiority of inclined over vertical web reinforcement regardless of the fact that these two types of reinforcement seem to be about equal for load-carrying capacity.

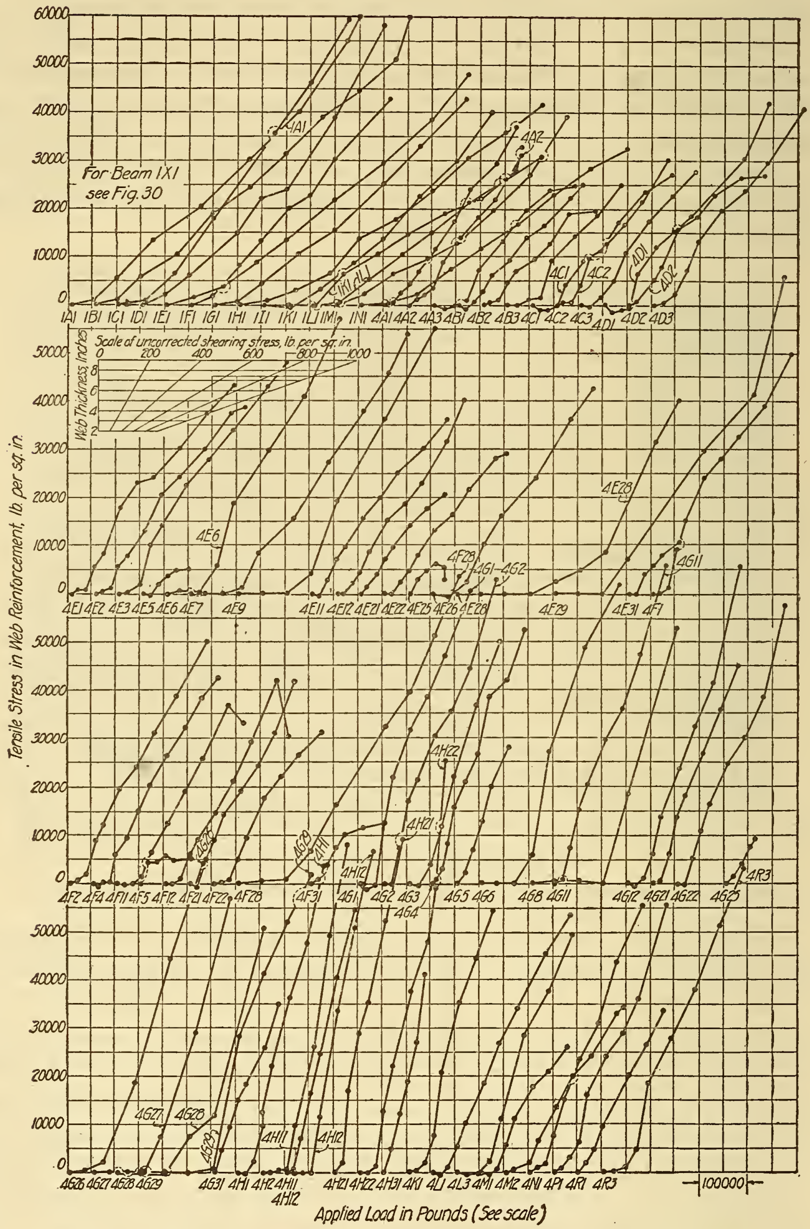

FIG. 74.—*Tensile stresses in web reinforcement for all beams, plotted against total load*

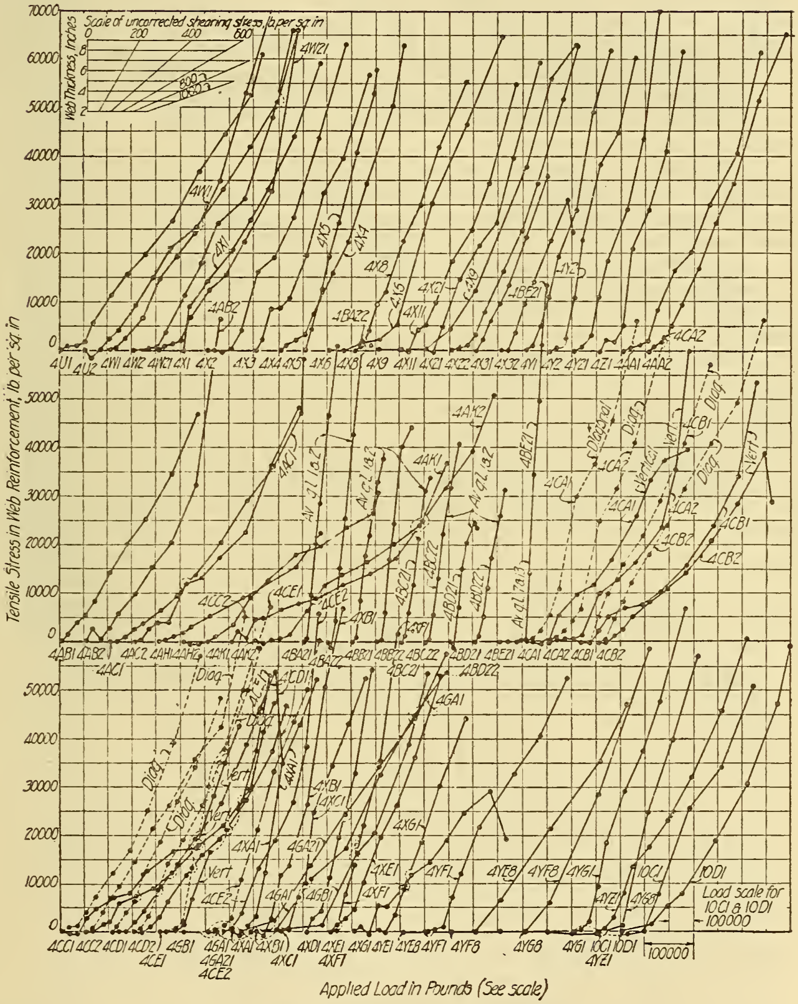

FIG. 75.—*Same as Figure 74*

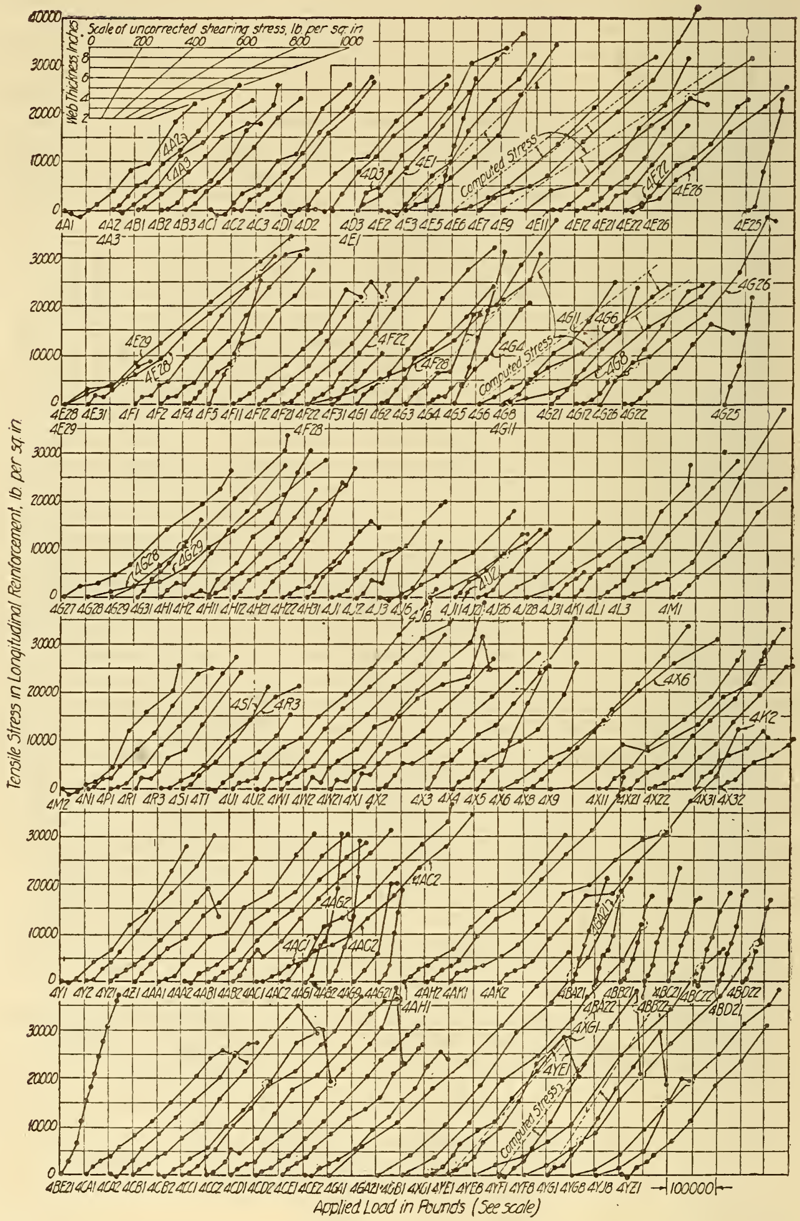

Fig. 76.—*Tensile stresses in longitudinal reinforcement for all beams, plotted against total load*

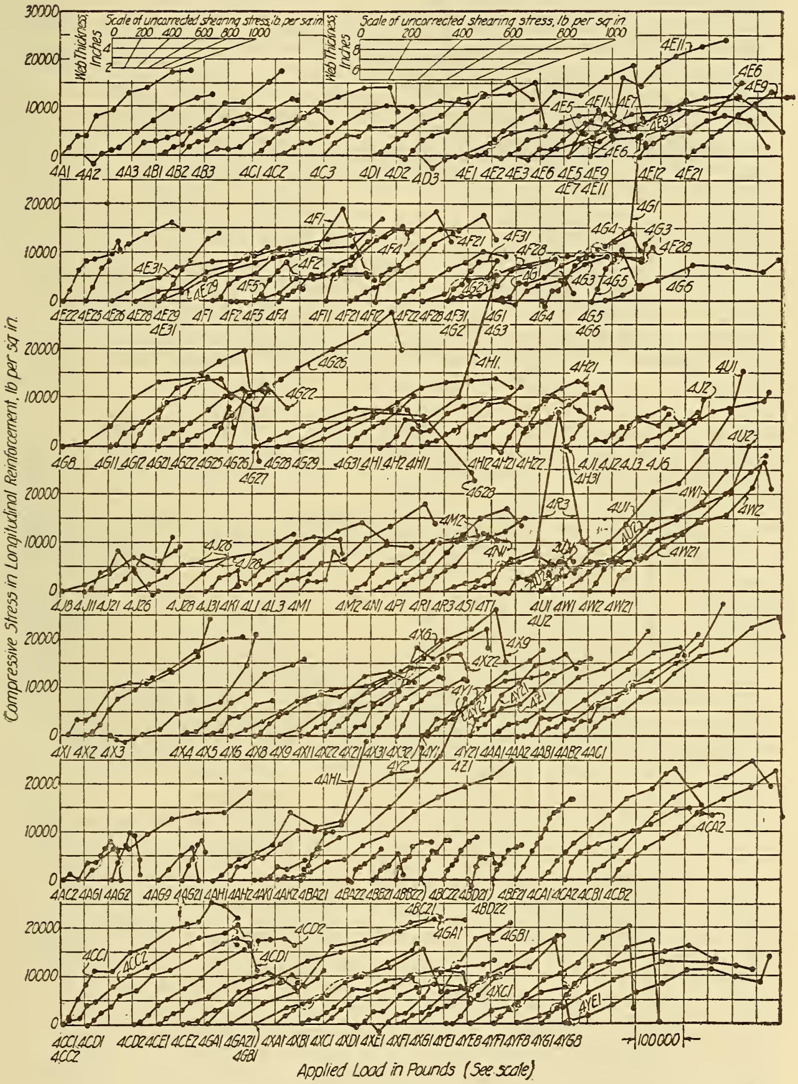

FIG. 77.—*Compressive stresses in longitudinal reinforcement for all beams, plotted against total load*

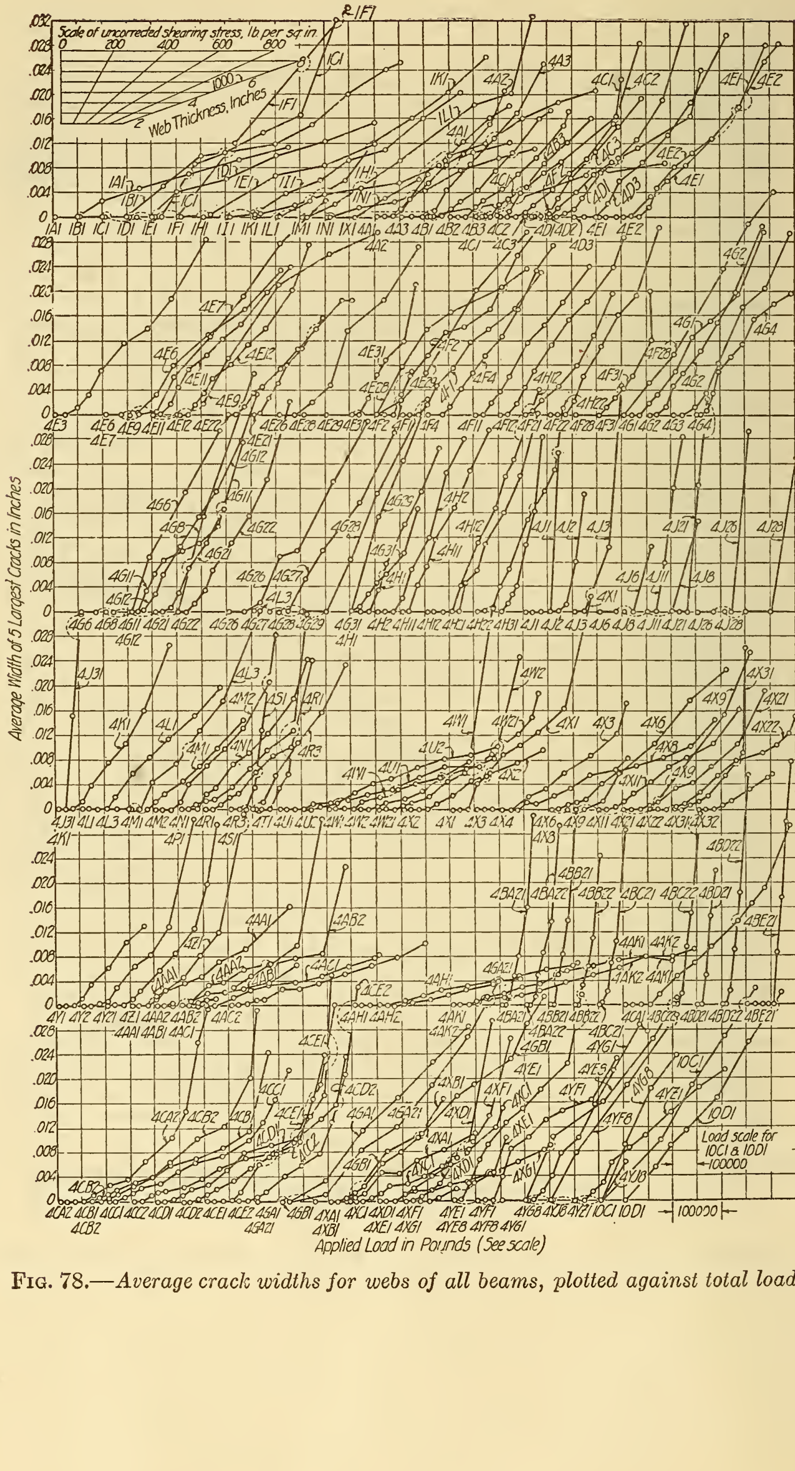

FIG. 78.—*Average crack widths for webs of all beams, plotted against total loads*

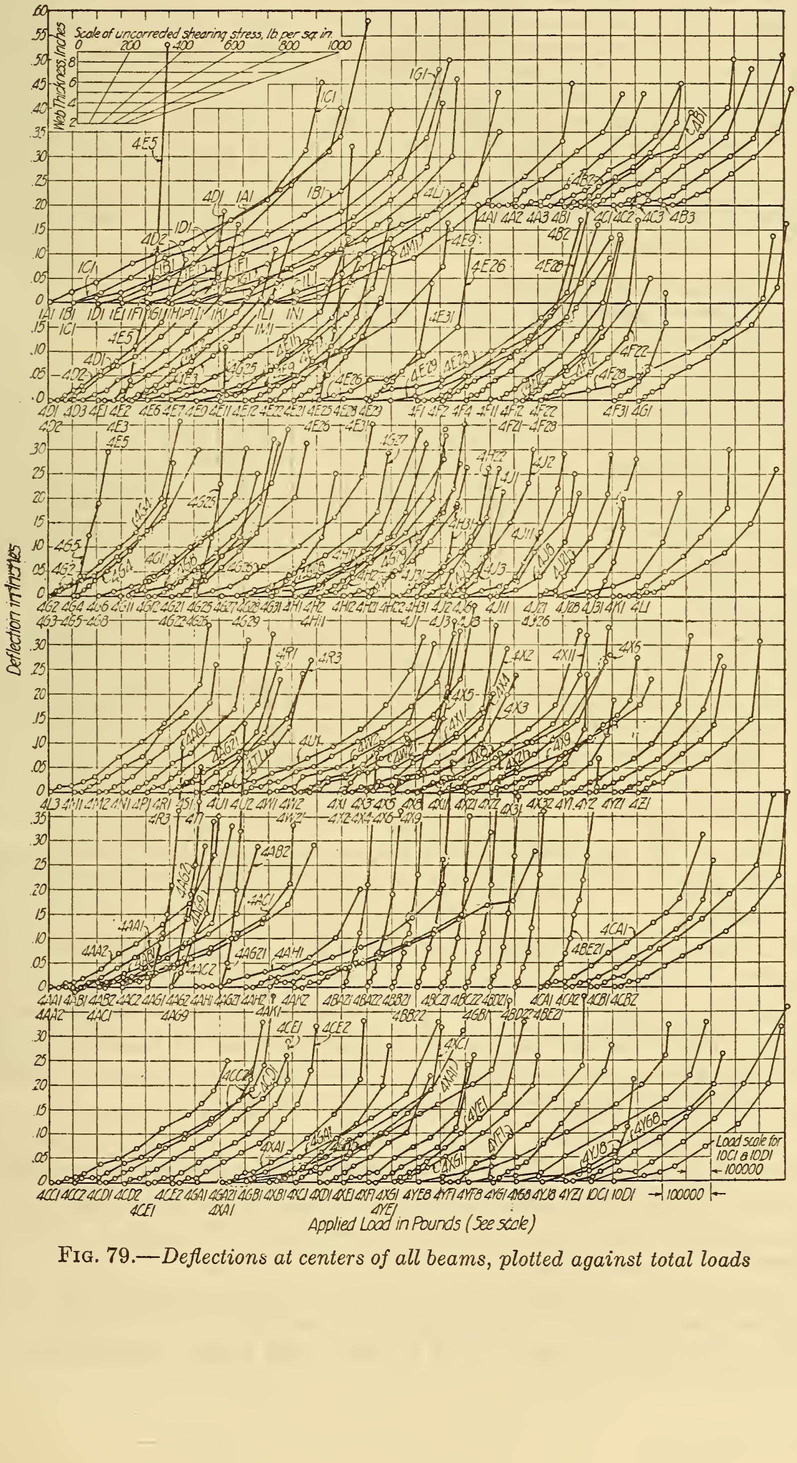

FIG. 79.—*Deflections at centers of all beams, plotted against total loads*

XXV. SUMMARY

In all, 172 beams were tested. Most of these were of I-shaped section. This shape was necessary in order to provide the necessary resistance to the horizontal, tensile, and compressive stresses and at the same time to permit the development of shearing stresses in the webs of 2,000 lbs./in.^{2} or greater. The web thickness varied from 2 to 12 inches. The majority of the beams had 3-inch webs. Those with 12-inch webs were made rectangular in cross section.

Most of the beams had a span of 9 feet 6 inches and a depth of 36 inches. Other spans and depths were:

1 beam, span 20 feet, depth 10 feet.
13 beams, span 16 feet, depth 4 feet 4 inches.
2 beams, span 9 feet 6 inches, depth 4 feet 0 inches.
9 beams, span 9 feet 6 inches, depth 1 foot 6 inches.

All the beams were heavily reinforced for longitudinal tension and most of them for longitudinal compression. The effort in the design was to force failure to occur in the web. This effort was successful with all except four beams. In two of these four beams the failure was by tension in the longitudinal reinforcement, and in the other two it was by crushing of the center pilaster upon which the load was applied.

The web reinforcement was in the form of stirrups in all cases except in 23 beams which had bars placed horizontally in the web and in 7 beams in which diamond mesh expanded metal was used as web reinforcement. Vertical stirrups, stirrups placed at 45° in the diagonal tension direction, and stirrups placed at 45° in the diagonal compression direction were used. Most of the beams had only one type of web reinforcement, tension bars placed vertically or at 45° in the direction of the diagonal tension. All the beams having diagonal compression reinforcement also had diagonal tension reinforcement, and all those having horizontal bars in the web also had vertical stirrups except 4S1 and 4T1. In a few cases vertical stirrups, stirrups placed at 45° in the tension direction, and stirrups placed at 45° in the compression direction were present in the same beam. The expanded metal web reinforcement was so placed that the long axis of the diamond was vertical and the strands of metal were inclined at about 22° on either side of the vertical.

The load was applied as a single load over a length of 8 to 12 inches of the flange. The ratio of the distance between the center of the load and the center of the reaction to the depth, d, varied from 1.03 to 3.8 and was 1.78 for the majority of the beams (36-inch beams).

Strain-gauge measurements to determine the stress in the web reinforcement and the longitudinal reinforcement and the strains in the concrete at all stages of the loading were made. Deflections at

the center of the span were measured for all beams. For a few beams deflections were measured at points in the span sufficiently close together to determine approximately the elastic curves of the beams.

Concretes having strengths ranging from about 2,100 to about 5,400 lbs./in.^{2} were used. In general, the reinforcing steel had a yield-point stress of about 60,000 lbs./in.^{2}. In a few cases bars of low yield point were inadvertently used.

To avoid confusion between bond failure and diagonal tension failure, the longitudinal bars and stirrups were anchored by means of hooks on the ends of the bars.

The heavy upper and lower concrete flanges, in conjunction with the center and end pilasters, formed a frame structure which apparently carried considerable load that did not pass down through the web of the beam as shear. Beams having no concrete or steel webs gave information on how much shear was in this way diverted from the webs. Accordingly, for the purpose of computing corrected shearing stress the loads for the beams with concrete webs were reduced by the amount of the load carried by the beams without webs when the center deflections in the two types of beams were the same.

The total shear reduced by the correction referred to above was assumed to be carried by the web of concrete and steel. Assuming that the longitudinal tensile and compressive stresses in any beam were proportional to their distances from the neutral axis, the maximum shear (that at the neutral axis) was computed by the usual formula

$$v=\frac{V}{b'jd}$$

in which V is the total shear less the shear carried by the flanges. Shearing stresses uncorrected for the resistance of the frames are also given.

The tests showed clearly that the tensile stress in the web reinforcement was independent of the compressive strength of the concrete in the web. This statement is based on measured stresses in beams having (1) concrete ranging in strength from 2,100 to 5,400 lbs./in.^{2}, (2) from 0.94 to 3.88 per cent of vertical web reinforcement, and (3) 1.71 per cent of web reinforcement inclined at 45° with the vertical.

In general, the shearing stresses (corrected to eliminate the effect of the stiffness of the heavy frames) developed in the beams tested may be expressed in terms of (1) the ratio, r, of web reinforcement, (2) the tensile stress, f_v, in the web reinforcement, and (3) the thickness, b', of the web, by the empirical equation

$$V=60+25b'+rf_v \tag{31}$$

71966°—26†——7

This equation indicates that with increasing web thicknesses there was an increasing effectiveness of the concrete in resisting shearing stress. This indication was found for loads causing tension in the web reinforcement up to 40,000 or 50,000 lbs./in.^{2}. It was not found at the maximum load, however, due in part at least to the fact that for the beams with the thickest webs the failure was by tension in the longitudinal reinforcement, and it is not known how great the shearing resistance was.

The increase in effectiveness of the concrete (in resisting shearing stresses) with increase of web thickness is probably only apparent, and due to the fact that with the thin webs a larger proportion of the thickness was occupied by the web bars than was the case with the thicker webs. For this reason and because of its greater simplicity the following formula for estimating the shearing strength of a beam seems preferable to formula (31):

$$v = (0.005 + r)f_v \tag{32}$$

This formula fits the test data reasonably well. It assumes that the concrete participated with the web reinforcement in resisting the shearing stresses, that it was equally effective for all thicknesses of web, and that the stresses carried by the concrete increased proportionally with increase in the shearing strength.

The smallest amount of web reinforcement used was about 0.5 per cent. The lack of test results for smaller percentages of web reinforcement and slender beams has been supplemented by results of tests carried out by the Deutscher Ausschuss für Eisenbeton. (See fig. 38.) These results agree quite well with those from the tests in this investigation. There are, however, some uncertainties in the interpretation of the data, since the stresses in the web reinforcement were not measured.

The presence of horizontal web bars in a beam which had vertical web bars appeared to have the effect of causing the stress in the vertical bars to be appreciably less than that in the vertical bars of a beam which had no horizontal bars. The reduction of the section of the web due to the presence of even the smallest (three-eighths inch) horizontal bars was so great, however, as to cause horizontal shear failures at loads lower than those carried by similar beams having no horizontal bars.

In beams having both vertical web bars and bars inclined in the tension direction at 45° with the vertical the maximum shearing stresses were about the same as those for beams in which the web reinforcement was all vertical or all inclined, and equal in amount to the sum of the vertical and inclined web reinforcement of the beams having both types of reinforcement. For web stresses below the yield point of the steel, however, the inclined bars developed greater stresses than the vertical bars in the same beam.

The presence of compression reinforcement in the web did not increase the shearing strength of the beams having such reinforcement. All of the beams, however, which had compression reinforcement in the web appeared to have sufficient compressive strength in the concrete of the web to resist all the compressive stresses in the webs without assistance from the reinforcement.

In the beams in which diamond mesh expanded metal was used as web reinforcement the presence of the compression strands did not appear to add to the shearing strengths of these beams. The tension strands appeared to have about the same value as an equal amount of reinforcement in the form of vertical or inclined bars.

Splicing the expanded metal by means of laps of about 5 inches along a vertical or a horizontal line developed stresses in the metal of more than 50,000 lbs./in.^{2} before failure occurred. They carried within 10 per cent of as great shearing stresses as did the corresponding beams in which the metal was not spliced.

One of the beams with expanded metal web reinforcement did not have the strands hooked around the longitudinal bars in the top and in the bottom of the beam. The anchorage was developed by bond and by the embedment of the bridges of metal in the concrete. This beam developed a tensile stress of only 45,000 lbs./in.^{2} in the metal before failure occurred, while those beams in which the strands were hooked around the longitudinal bars developed stresses of from 50,000 to 60,000 lbs./in.^{2}. The shearing stress at failure was also correspondingly less than that for beams with hooked reinforcement.

High diagonal compressive stresses were developed in many of the beams. Failure by diagonal compression occurred only with beams having vertical web reinforcement. The analysis given indicates that diagonal compressive stresses are twice as great for a given shear in beams with vertical stirrups as for a beam with stirrups placed at 45° with the vertical. The tests indicate that approximately this relation exists. The data, however, are not conclusive.

For most of the beams the distance between stirrups at right angles to their direction was from about one-eighth to about one-tenth of the depth of the beam. In a few beams the spacing of the stirrups was varied for the purpose of studying the effect on the shearing strength of the beam. There was a tendency toward a falling off in shearing strength with increase in spacing of stirrups. The test data are insufficient, however, to show at what spacing the effectiveness of the stirrups begins to decrease.

All stirrups were anchored by means of hooks in the top and in the bottom of the beam. In all cases the hooks passed around the longitudinal tension bars in the bottom of the beam and around longitudinal compression bars in the top of the beam. In a number of cases the horizontal bars in the top of the beam were not needed

for resistance to compression and were used merely for the purpose of holding the stirrups in position. They were very useful for this purpose, but there was no indication that they were necessary as a part of the anchorage of the stirrups. Proper embedment in the concrete in the bottom of the beam appeared to give sufficient anchorage to all except some of the largest bars used. The effectiveness of the anchorage appeared to be somewhat less for the thin webs than for the thick webs. Rigid connection to the longitudinal tension bars was not essential for effectiveness of either vertical or inclined bars.

All longitudinal tension bars were 1¼ inches in diameter and were anchored, usually by means of a semicircular hook having a radius of 7.5 inches for the lower layer and of 5 inches for the upper layer of tension bars. The hooks for the upper layer fitted inside the hooks for the lower layer of bars.

In spite of the anchorage there was evidence of slipping of the bars. Stresses close to the support of more than half those at the center of the span indicate the necessity for anchorage of the bars.

The data on the effect of depth of beam on web stresses and web resistance were not extensive and other variables than depth of beam were involved, so that the results are not conclusive. Except for beam 4BE21,, which was rectangular in cross section, the equation $v = (0.005 + r) f_v$, which was derived from the data of the beams 36 and 52 inches deep, seems to apply fairly well to the data of the beams which were 18 inches deep.

The shearing stress at which the first diagonal crack in the web of the beam occurred varied with the thickness of the web and the compressive strength of the concrete. It was nearly independent of the amount, spacing, and direction of the web reinforcement. The shearing stress at first crack varied in general from about 100 to about 300 lbs./in.^{2}. A few cases were outside these limits. The rate of widening of the cracks was independent of the amount of horizontal web reinforcement and the compressive strength of the concrete. It was slightly dependent upon the amount and spacing of vertical web reinforcement and upon the thickness of the web. The rate of widening was much less for beams with inclined than for beams with vertical stirrups.

The presence of web stiffeners had some effect in causing the earliest web cracks to take a steeper slope than they took in beams without stiffeners. The distance between stiffeners was less than the depth of the beams. Before failure occurred cracks crossed through the stiffeners, and the direction of the later cracks seemed to be independent of the presence of web stiffeners.

The presence of web stiffeners was in general without marked effect on the behavior of the beam after the formation of the earliest cracks.

In beam 1X1, however, which had a depth of 10 feet, a span of 20 feet, and a web thickness of 3.9 inches, the failure in vertical shear was limited to the space between two adjacent stiffeners. It is probable that the stiffeners were of some effect in so limiting the area involved in the failure.

The amount of the deflections appeared to be more dependent upon the shearing stresses and the width of cracks than upon the horizontal tensile and compressive stresses in the top and bottom flanges. The deflections were generally greater for beams with vertical than for beams with inclined web reinforcement. For equal shearing stresses the deflections were greater for beams with thick than for beams with thin webs.

TABLE 6.—*Data of series 1 and 10*

Reference No.			115	116	117	118	119	120	121	122	123	124
Beam No.			1A-C-D1	1B-E-H1	1F1	1G1	1I1	1K1-L1	1M1	1N1	1X1	10C1-D1
Web thickness, in.			4.55	4.30	4.25	3.40	4.10	4.05	5.05	4.45	3.90	8.10
Flange width - in.			24.30	24.10	24.05	23.60	24.05	24.05	24.75	24.25	28.00	20.05
Web reinforcement	Vertical bars	Diam., in.	None	None	3/4	None	3/4	3/4	3/4	3/4	3/4	1/2
		Ratio	0		.0173		.0178	.0268	.0145	.0199	.0262	.0143
	Horizontal bars	Diam., in.	None	None	None	None	1/2	None	3/4	3/4	3/4	1/2
		Ratio	0		0		.0030	0	.0145	.0199	.0189	.0060
	Inclined bars	Diam., in.	1/2	3/4	None	1/2	None	None	None	None	None	None
		Ratio	.0154	.0244	0	.0206	0	0	0	0	0	0
	Spacing, in.		2.83	4.25	6	2.83	6 (15)	4	6 (15)	5	4.5(6)	3.33(8)
No. of 1¼-in. flange bars	Top		17	17	17	17	17	17	17	17	20	12
	Bottom		17	17	17	17	17	17	17	17	20	12
Age at test, days			34	33	28	29	29	31	30	30	44	37
Cylinder strength	At 7 da. lb./in.²											2700
	At test of beam		2410	2930	1490	1800	4080	3770	3280	4050	4620	5050
Manner of failure			D.T.	D.T.	D.C.	D.T.	D.C.	D.C.	D.C.	D.C.	V.S.	C.P.
Maximum applied load, lb.			553700	663300	414000	600000	505000	503000	475000	568000	1363000	706000
Shearing stress on gross section at tensile stress in web reinf. of:		10000	350	490	310	550	550	490	430	560	690	380
		20000	580	810	530	940	820	890	720	900	1230	530
		30000	790	1120	770	1280	1040	1250	910	1240		670
		40000	980	1380	1000	1590	1280					790
		50000	1140	1640		1910						900
		60000	1290									1020
Shearing stress at maximum load based on:	Gross section		1340	1700	1070	2240	1350	1360	1030	1400	1540	1030
	Net vert. "				1300		1660	1660	1210	1690	1910	1180
	Net horiz. "						1540		1210	1690	1910	1180
Ten. stress at max. load	Web reinforcem't		Y.P.	53000	43000	51000	43500	32000	39000	35000	26000	63000
	Longitudinal "											
Deflection (inches) at uncorrected shearing stress of:		200	.029	.021	.025	.005	.026	.022	.020	.022	.024	.017
		400	.069	.046	.053	.030	.054	.060	.066	.057	.060	.044
		600	.114	.075		.054	.086	.103	.121	.100	.109	.092
		800	.167	.108		.078	.135	.150	.185	.148	.159	.158
		1000	.228	.141		.113	.190	.206	.304	.208	.226	
		1200	.332	.180		.148	.254			.273	.287	
		1600		.330		.235						
Tensile stress in web reinf. when av. width of 5 largest cracks was:		.01 in.	27100	29400	8500		16000	15000	31500	13800	8600	7900
		.02 in.	54500		20500		39800	25500		30000	21700	22600
Average width of 5 largest cracks at uncorrected shearing stress of:		200	.0010	.0000	.0070		.0011	.0001	.0009	.0010	.0007	.0042
		400	.0047	.0018	.0148		.0045	.0036	.0026	.0047	.0070	.0126
		600	.0084	.0046	.0223		.0078	.0079	.0052	.0083	.0100	.0215
		800	.0110	.0075	.0299		.0113	.0127	.0035	.0121	.0122	.0300
		1000	.0133	.0103	.0376		.0147	.0179	.0107	.0157	.0139	.0387
		1200	.0260		.0000		.0188	.0224		.0195	.0187	
		1600										
Shearing stress when av. width of 5 largest cracks was:		.01 in.	750	1100	280		720	690	930	690	600	330
		.02 in.	1100		550		1250	1100		1240	1330	560

Note: All stresses are given in pounds per square inch.

TABLE 7.—*Descriptive data of test beams*

NOTE: All longitudinal flange bars were 1¼" in diameter.

Reference No.	Beam No.	Web thickness	Flange width	Web Reinforcement							No. of 1¼-in. flange bars		Age at test	Cylinder strength[A]		Manner of failure
				Vertical Bars		Horizontal Bars		Inclined Bars		Spacing						
				Diam.	Ratio	Diam.	Ratio	Diam.	Ratio		Top	Bottom		At 7 da.	At test of beam	
		in.	in.	in.		in.		in.		in.			da.	lb./in.²	lb./in.²	
1	4A1-2	3.10	12.05	3/4	.0354	3/4	.0354	0	0	4	8	8	47	3380	4910	H.S.
2	4B1-2	3.25	12.05	3/4	.0340	5/8	.0236	0	0	4	8	8	47	2970	5390	H.S.
3	4C1-2	3.25	12.20	3/4	.0340	1/2	.0151	0	0	4	8	8	46	3170	4770	H.S.
4	4D1-2	3.15	12.20	3/4	.0350	3/8	.0087	0	0	4	8	8	60	3810	5810	H.S.
5	4E1-2	3.30	12.30	3/4	.0335	0	0	0	0	4	8	8	60	3670	5940	D.T.
6	4YE1	2.65	12.00	3/4	.0416	0	0	0	0	4	1	7	60	3070	5700	V.S.
7	4A3	4.30	12.35	3/4	.0257	3/4	.0257	0	0	4	8	8	60	2570	5000	H.S.
8	4B3	4.25	12.30	3/4	.0260	5/8	.0180	0	0	4	8	8	60	2380	5670	H.S.
9	4C3	4.20	12.30	3/4	.0263	1/2	.0117	0	0	4	8	8	62	2940	5980	H.S.
10	4D3	4.50	12.35	3/4	.0246	3/8	.0061	0	0	4	8	8	60	2640	5580	D.T.
11	4E3	4.45	12.30	3/4	.0248	0	0	0	0	4	8	8	59	3040	5400	D.T.
12	4E6-7	5.90	15.20	3/4	.0188	0	0	0	0	4	10	10	58	3320	5340	
13	4E9	8.55	17.75	3/4	.0130	0	0	0	0	4	12	12	60	3570	5970	D.T.
14	4YE8	8.35	17.20	3/4	.0133	0	0	0	0	4	2	12	61	3350	5780	D.T.
15	4E11-12	2.95	12.05	3/4	.0375	0	0	0	0	4	8	8	59	2410	4610	V.S.
16	4E21-2	2.65	12.00	3/4	.0413	0	0	0	0	4	8	8	59	1680	4550	V.S.
17	4E26	5.95	15.20	3/4	.0186	0	0	0	0	4	10	10	59	1650	4000	D.T.
18	4E28-9	8.45	17.80	3/4	.0130	0	0	0	0	4	12	12	59	1620	3510	D.T.
19	4E31	2.55	11.85	3/4	.0434	0	0	0	0	4	8	8	60	790	2180	V.S.
20	4M1-2	3.25	12.25	5/8	.0234	5/8	.0234	0	0	4	8	8	61	3040	5180	H.S.
21	4YZ1	2.85	12.05	5/8	.0269	0	0	0	0	4	8	8	59	3100	6040	
22	4F1-2	3.05	12.15	5/8	.0251	0	0	0	0	4	8	8	62	3700	5700	D.T.
23	4YF1	2.95	11.90	5/8	.0260	0	0	0	0	4	1	6	59	3240	5580	D.T.
24	4F4	2.30	12.30	5/8	.0334	0	0	0	0	4	8	8	60	3350	5550	V.S
25	4YF8	8.35	17.65	5/8	.0092	0	0	0	0	4	2	12	59	3210	6080	D.T.
26	4F11-12	2.75	11.95	5/8	.0280	0	0	0	0	4	8	8	59	2430	4350	V.S.
27	4F21-2	2.90	12.10	5/8	.0264	0	0	0	0	4	8	8	59	1590	3470	V.S.
28	4F28	8.35	17.60	5/8	.0092	0	0	0	0	4	12	12	59	1800	3690	D.T.
29	4F31	2.75	11.95	5/8	.0279	0	0	0	0	4	8	8	60	990	1950	V.S.
30	4N1	2.95	12.10	1/2	.0166	3/8	.0094	0	0	4	8	8	59	2950	4630	H.S.
31	4L1	3.40	12.20	1/2	.0144	1/2	.0144	0	0	4	8	8	63	3560	5400	H.S.
32	4P1	3.35	12.20	1/2	.0146	5/8	.0229	0	0	4	8	8	61	3120	4980	H.S.
33	4R1	3.25	12.20	1/2	.0151	3/4	.0340	0	0	4	8	8	62	3440	5000	H.S.
34	4G1-2	3.15	12.15	1/2	.0155	0	0	0	0	4	8	8	60	3460	5550	D.T.
35	4YG1	2.95	12.10	1/2	.0166	0	0	0	0	4	1	5	59	2920	5160	D.T.
36	4L3	4.30	12.20	1/2	.0114	1/2	.0114	0	0	4	8	8	59	2980	5480	D.T.
37	4R3	4.10	12.10	1/2	.0120	3/4	.0270	0	0	4	8	8	60	2640	5050	H.S.
38	4G3	4.10	12.05	1/2	.0120	0	0	0	0	4	8	8	59	3020	5750	D.T.
39	4G4	1.95	12.15	1/2	.0252	0	0	0	0	4	8	8	59	3540	6200	V.S.
40	4G6	5.95	15.10	1/2	.0083	0	0	0	0	4	10	10	59	3550	5720	D.T.
41	4G8	8.35	17.50	1/2	.0059	0	0	0	0	4	12	12	60	3390	5700	D.T.
42	4YG8	8.45	17.70	1/2	.0058	0	0	0	0	4	2	12	59	3360	5010	D.T.
43	4GA1	12.00	12.00	1/2	.0082	0	0	0	0	2	8	8	60	3170	4880	L.T.
44	4G11-12	2.75	12.05	1/2	.0177	0	0	0	0	4	8	8	60	2410	4720	D.T.
45	4G21-2	2.75	12.15	1/2	.0178	0	0	0	0	4	8	8	60	1460	3650	V.S.
46	4G26-7	5.65	14.90	1/2	.0086	0	0	0	0	4	10	10	60	1610	4150	D.T.
47	4G28-9	8.20	17.55	1/2	.0060	0	0	0	0	4	12	12	60	1650	4000	D.T.
48	4GA21	12.10	12.10	1/2	.0081	0	0	0	0	2	8	8	59	1500	3000	L.T.
49	4G31	2.70	12.10	1/2	.0182	0	0	0	0	4	8	8	60	970	2820	V.S.
50	4GB1	12.10	12.10	3/8	.0088	0	0	0	0	1	8	8	61	3410	5960	
51	4K1	3.10	12.10	3/8	.0089	3/8	.0089	0	0	4	8	8	62	3600	5250	H.S.
52	4H1-2	3.40	12.35	3/8	.0081	0	0	0	0	4	8	8	61	3380	5920	D.T.
53	4H11-12	2.85	12.10	3/8	.0095	0	0	0	0	4	8	8	59	2200	5170	D.T.
54	4H21-2	2.70	12.05	3/8	.0100	0	0	0	0	4	8	8	59	1380	3990	
55	4H31	2.60	11.95	3/8	.0106	0	0	0	0	4	8	8	60	610	1520	D.C.
56	4BA21-2	2.60	12.00	3/8	.0116	0	0	0	0	3.62	4	4	60	1410[E]	4170[E]	D.T.
57	4BB21-2	2.60	12.10	1/2	.0115	0	0	0	0	6.5	4	4	60	1630[E]	4030[E]	D.T.

[A] Cylinders 8"×16" except as noted. [E] 6"×12" cylinder.

TABLE 8.—*Maximum loads, shearing stresses at varying tensile stresses, and tensile stresses at maximum load*

NOTE: Stresses are given in pounds per square inch.

Reference No.	Beam No.	Maximum applied load	Shearing stress at various loads									Tensile stress at maximum load in:	
			Shearing stress on gross section at tensile stress in web reinforcement of:						Shearing stress at maximum load based on:				
		lb:	10000	20000	30000	40000	50000	60000	Gross section[F]	Net vert. section[H]	Net horiz. section[K]	Web reinforcement	Longitudinal reinforcement
1	4A1-2	269000	470	850	1320				1460	1930	1930	36500	24000
2	4B1-2	252100	590	1170					1320	1710	1630	22500	24000
3	4C1-2	293600	530	960					1530	1990	1810	34500	27000
4	4D1-2	232400	480	930					1750	2310	1990	39000	28000
5	4E1-2	338900	440	750	1140				1750	2270	1750	42500	34500
6	4YE1	266000	420	950	1550				1700	2370		33000	33000
7	4A3	293100	460	780					1160	1400	1400	31000	22500
8	4B3	266500	410	800					1060	1290	1250	28000	25500
9	4C3	299900	370	670	1050				1210	1470	1370	34000	33000
10	4D3	342200	320	560	920	1150			1290	1550	1410	46000	37000
11	4E3	352400	270	490	810	1100			1340	1620	1340	48000	40500
12	4E6-7	433100	400	560	750	940	1090		1250	1430		Y.P.	37000
13	4E9	519700	310	420	530	640	750	870	1030	1130		Y.P.	32000
14	4YE8	516700	460	590	740	870	950	1040	1050	1150			
15	4E11-12	275900	430	850	1300				1580	2130		38000	29000
16	4E21-2	218000	520	1080					1380	1910		26000	20500
17	4E26	332000	310	490	690	890			950	1080		43000	25500
18	4E28-9	487400	-480	600	710	850			970	1070		Y.P.	34000
19	4E31	132500	640						880	1250		12500	9000
20	4M1-2	252700	410	710	1100				1300	1610	1610	33500	24000
21	4YZ1	306000[A]	400	740	1100	1370	1600	1800	1820[A]	2340[A]		61000[A]	31000[A]
22	4F1-2	321800	330	560	920	1310	1600		1800	2270		59500	37000
23	4YF1	295000	370	590	910	1330	1600		1700	2150		Y.P.	38500
24	4F4	273000	550	870	1310	1790			2020	2770		48000	31500
25	4YF8	494200	450	540	630	700	770	840	1000	1080			
26	4F11-12	258800	530	880	1180	1470			1600	2070		44500	28000
27	4F21-2	222900	380	700	1130				1300	1660		36500	25000
28	4F28	481400	350	460	590	730	810	900	980	1050		Y.P.	31000
29	4F31	108000	440						670	860		15000	10500
30	4N1	260700	340	530	750	950	1200	1480	1500	1810	1720	Y.P.	27000
31	4L1	286500	350	560	780	1040	1320		1430	1680	1680	53500	28500
32	4P1	262000	310	400	760	960	1100	1240	1330	1560	1630	Y.P.	27500
33	4R1	221000	300	550	840	1060			1150	1360	1500	33500	26000
34	4G1-2	288300	240	360	550	850	1050	1240	1550	1840		Y.P.	30000
35	4YG1	248200	320	440	640	820	1050	1240	1430	1720		Y.P.	32000
36	4L3	331000	370	470	590	790	960	1120	1310	1480	1480	Y.P.	39000
37	4R3	289100	340	430	640	820	980	1150	1200	1360	1460	Y.P.	24000
38	4G3	316000	270	370	500	640	780	930	1310	1490		Y.P.	38000
39	4G4	216200	340	590	900	1270	1600		1880	2530		57500	21000
40	4G6	394300	330	380	430	530	630	790	1120	1230	Y.P.	Y.P.	24500
41	4G8	431600	360	420	480	540	600	650	880	930		Y.P.	25000
42	4YG8	428000	450	490	550	610	680	730	860	910			
43	4GA1	496000	300	430	490	570	660		700	760		53500	Y.P.
44	4G11-12	258700	320	500	730	1000	1170	1340	1580	1920		Y.P.	25500
45	4G21-2	201100	290	510	800	1050			1240	1510		51000	22500
46	4G26-7	354300	380	490	580	670	760		1060	1160		Y.P.	35000
47	4G28-9	426500	360	420	480	540	620	690	880	930			
48	4GA21	496200	270	360	460	560	640		700	760		57500	Y.P.
49	4G31	138300	240	500	760				870	1070		35000	16000
50	4GB1	540000	470	530	590	640			750	780			
51	4K1	240000	320	390	520	690	890	1200	1310	1490	1490	Y.P.	27500
52	4H1-2	259100	320	400	500	620	730	840	1290	1460	1290	Y.P.	29000
53	4H11-12	238000	340	440	550	680	810		1410	1630		Y.P.	24000
54	4H21-2	199100	230	330	460	620	770		1230	1430		Y.P.	20500
55	4H31	88800	200	370	490	570			580	680		41000	9500
56	4BA21-2	69900	230	370	500	620	750	870	880	1030		Y.P.	21000
57	4BB21-2	69800	250	380	570				890	1100		44000	21000

[A] Not carried to failure. [F] Based on total web thickness, b'. [H] Based on b' minus diameter vertical web bars. [K] Based on b' minus diameter horizontal web bars.

TABLE 7.—*Descriptive data of test beams*—Continued

NOTE: All longitudinal flange bars were 1¼" in diameter.

Reference No.	Beam No.	Web thickness	Flange width	Web Reinforcement						Spacing	No. of 1¼-in. flange bars		Age at test	Cylinder strength[A]		Manner of failure
				Vertical Bars		Horizontal Bars		Inclined Bars								
		in.	in.	Diam. in.	Ratio	Diam. in.	Ratio	Diam. in.	Ratio	in.	Top	Bottom	da.	At 7 da. lb/in.^{2}	At test of beam lb/in.^{2}	
58	4BC21-2	2.45	11.90	5/8	.0124	0	0	0	0	10	4	4	60	1520	3910[E]	D.T.
59	4BD21	2.50	11.95	3/4	.0098	0	0	0	0	18	4	4	61	1690	3800[E]	D.T.
60	4BD22	2.40	12.00	3/4	.0128	0	0	0	0	14.38	4	4	59	1740	4510[E]	D.T.
61	4BE21	11.80	11.80	3/8	.0023	0	0	0	0	4	4	4	63	1210	3600[E]	D.T.
62	4CA1-2	2.90	12.10	5/8	.0186	0	0	3/8	.0070	5	8	8	59	3570	5640	V.T.
63	4CB1-2	2.90	12.15	5/8	.0186	0	0	1/2	.0122	5	8	8	59	3320	5910	V.T.
64	4CC1-2	2.80	12.00	5/8	.0193	0	0	5/8	.0193	5	8	8	59	3420	5940	V.T.
65	4CD1-2	2.70	12.00	1/2	.0131	0	0	1/2	.0131	5	8	8	60	3020	5470	
66	4CE1-2	2.80	12.05	3/8	.0070	0	0	3/8	.0070	5	8	8	60	3790	5950	D.T.
67	4U1-2	3.10	12.15	0	0	0	0	3/4	.0354	4	8	8	59	3200	5120	
68	4AC1-2	3.15	12.15	0	0	0	0 F	3/4	.0351	4	8	8	60	3140	5530	Slip
69	4W1-2	3.20	12.20	0	0	0	0	5/8	.0238	4	8	8	59	3630	5500	D.T.
70	4AB1-2	3.05	12.05	0	0	0	0 F	5/8	.0252	4	8	8	59	3130	5310	D.T.
71	4W21	2.85	12.00	0	0	0	0	5/8	.0269	4	8	8	59	1370	3650	D.T.
72	4X1-2-8	3.10	12.15	0	0	0	0	1/2	.0158	4	8	8	59	3350	5400	D.T.
73	4AA1-2	3.30	12.30	0	0	0	0 F	1/2	.0149	4	8	8	59	2930	5190	D.T.
74	4X3	4.25	12.10	0	0	0	0	1/2	.0116	4	8	8	60	3220	4380	D.T.
75	4X4	2.05	11.90	0	0	0	0	1/2	.0240	4	8	8	59	3300	4650	D.T.
76	4X6	5.90	15.20	0	0	0	0	1/2	.0083	4	10	10	59	3400	5950	D.T.
77	4X9	8.40	17.60	0	0	0	0	1/2	.0058	4	12	12	59	3520	4990	D.T.
78	4X11	2.95	12.15	0	0	0	0	1/2	.0167	4	8	8	60	2420	5160	D.T.
79	4X21-2	2.85	12.10	0	0	0	0	1/2	.0172	4	8	8	60	1710	3520	D.T.
80	4X31-2	2.55	12.10	0	0	0	0	1/2	.0191	4	8	8	60	910	2270	
81	4Y1-2	3.20	12.15	0	0	0	0	3/8	.0087	4	8	8	60	3510	5230	D.T.
82	4Z1	3.30	12.20	0	0	0	0 F	3/8	.0084	4	8	8	59	3690	4380	D.T.
83	4Y21	2.95	12.00	0	0	0	0	3/8	.0094	4	8	8	59	1600	3240	D.T.
84	4AH1-2	3.30	12.20	0	0.	0	0	1	.0595	4	8	8	59	3380	5200	Slip
85	4AK2	3.15	12.15	0	0	0	0 F	1	.0624	4	8	8	60	3430	5950	Slip
86	4AK1	4.15	12.95	0	0	0	0 F	1	.0474	4	8	8	60	3400	4710	Slip
87	4XG1	3.25	12.10	Expanded metal web reinf. Strands inclined in compression direction neglected in computing ratio of reinforcement.					.0192		8	8	58	2900	4770	V.S.
88	4XA1	3.45	12.35						.0128		8	8	60	3580	5080	D.T.
89	4XB1	2.95	12.20						.0106		8	8	60	3170	6190	D.T.
90	4XC1	3.20	12.25						.0056		8	8	60	2780	5170	V.S.
91	4XD1	3.20	12.15						.0188		8	8	58	2960	5320	D.T.
92	4XE1	3.05	12.00						.0146		8	8	58	2710	4970	D.T.
93	4XF1	3.00	12.00						.0148		8	8	61	3000	5600	D.T.
94	4E5	0	12.10	3/4		0		0		4	8	8	59	3390	6140	D.T.
95	4E25	0	12.30	3/4		0		0		4	8	8	60	1470	3490	D.T.
96	4F5	0	12.00	5/8		0		0		4	8	8	59	3180	6190	D.T.
97	4G5	0	12.40	1/2		0		0		4	8	8	60	3250	6070	D.T.
98	4G25	0	12.20	1/2		0		0		4	8	8	60	1700	4070	D.T.
99	4X5	0	12.05	0		0		1/2		4	8	8	59	3280	4460	D.T.
100	4S1	3.40	12.20	0	0	1/2	.0144	0	0	4	8	8	60	3420	4600	D.T.
101	4T1	3.40	12.15	0	0	3/4	.0324	0	0	4	8	8	60	3680	5120	D.T.
102	4J1-2	3.20	12.25	0	0	0	0	0	0		8	8	62	3310	5380	D.T.
103	4J11	2.70	12.05	0	0	0	0	0	0		8	8	59	2080	4910	D.T.
104	4J21	3.10	12.25	0	0	0	0	0	0		8	8	60	1740	4080	D.T.
105	4J31	2.75	11.95	0	0	0	0	0	0		8	8	60	560	1570	D.T.
106	4J3	4.70	12.80	0	0	0	0	0	0		8	8	59	2780	5670	D.T.
107	4J6	6.05	15.25	0	0	0	0	0	0		10	10	59	3500	6180	D.T.
108	4J26	5.75	15.05	0	0	0	0	0	0		10	10	59	1670	3650	D.T.
109	4J8	8.55	17.70	0	0	0	0	0	0		12	12	59	3040	6050	D.T.
110	4YJ8	8.35	17.60	0	0	0	0	0	0		0	5	60	3200	5500	D.T.
111	4J28	8.50	17.60	0	0	0	0	0	0		12	12	60	1490	4060	D.T.
112	4AG1-2	0	12.30	0		0		0			8	8	59	3490	5700	D.T.
113	4AG21	0	12.25	0		0		0			8	8	60	1560	3910	D.T.
114	4AG9	0	17.60	0		0		0			12	12	59	2970	6200	D.T.

[A] Cylinders 8"×16" except as noted. [E] 6"×12" cylinders. [F] Inclined compression bars same size as inclined tension bars.

TABLE 8.—*Maximum loads, shearing stresses at varying tensile stresses, and tensile stresses at maximum load*—Continued

NOTE: Stresses are given in pounds per square inch.

Reference No.	Beam No.	Maximum applied load	Shearing stress at various loads									Tensile stress at maximum load in:	
			Shearing stress on gross section at tensile stress in web reinforcement of:						Shearing stress at maximum load based on:				
		lb.	10000	20000	30000	40000	50000	60000	Gross section[F]	Net vert. section[H]	Net horiz. section[K]	Web reinforcement	Longitudinal reinforcement
58	4BC21-2	65400	320	460	690				880	1170		37500	18500
59	4BD21	56100	330	550					750	1070		26000	18500
60	4BD22	62000	350	590	840				860	1250		31000	18000
61	4BE21	121300	180	190	200	210	230	250	340	350		Y.P.	37000
62	4CA1-2	322600	400[A]	540[A]	700[A]	960[A]	1140[A]	1290[A]	1870	2370		Y.P.	34500
63	4CB1-2	368200	390[A]	730[A]	1040[A]	1330[A]	1600[A]	1800[A]	2130	2710		Y.P.	40500
64	4CC1-2	381100	520[A]	880[A]	1210[A]	1530[A]	1770[A]	1890[A]	2280	2940		Y.P.	37500
65	4CD1-2	347300	380[A]	690[A]	990[A]	1250[A]	1490[A]	1690[A]	2160	2650		Y.P.	35500
66	4CE1-2	297000	400[A]	570[A]	730[A]	960[A]	1150[A]	1270[A]	1780	2060		Y.P.	25000
67	4U1-2	426500	580	970	1420	1730	2090	2270	2320			64000	41000
68	4AC1-2	400500	700	1170	1500	1790	2000		2150			Y.P.	35500
69	4W1-2	354000	500	790	1070	1310	1510	1660	1860			Y.P.	35000
70	4AB1-2	330500	570	900	1110	1400	1550	1650	1840			Y.P.	29500
71	4W21	329500	550	1000	1310	1460	1560	1660	1960			Y.P.	36000
72	4X1-2-8	326800	350	630	890	1110	1330	1500	1790			Y.P.	30500
73	4AA1-2	336600	390	660	870	1100	1250	1420	1730			Y.P.	30500
74	4X3	306300	240	440	580	730	830	970	1220			Y.P.	35500
75	4X4	258000	690	1030	1350	1650	1930	2090	2130			63000	26000
76	4X6	448800	450	540	620	750	870	990	1290			Y.P.	31000
77	4X9	481600	380	470	550	630	690	870	970			Y.P.	33000
78	4X11	299000	410	660	910	1120	1280	1430	1720			Y.P.	28000
79	4X21-2	296200	410	660	910	1130	1300	1460	1760				
80	4X31-2	150100	370	630	880				990			30000	11000
81	4Y1-2	264900	270	370	520	640	800	940	1400			Y.P.	29000
82	4Z1	252800	330	450	570	720	820	910	1300			Y.P.	25500
83	4Y21	227500	300	440	630	770	870	950	1310			Y.P.	21000
84	4AH1-2	372600	890	1570					1920			26500	36000
85	4AK2	493000	720	1520	2060	2440	2620		2660			37000	41000
86	4AK1	446000	800	1480	1710				1820			51000	52000
87	4XG1	299400	580	780	960	1150	1350	1420	1560			Y.P.	28500
88	4XA1	296300	330	440	550	680	840	1030	1460			Y.P.	27000
89	4XB1	247500	330	510	650	740	820	940	1420			Y.P.	27500
90	4XC1	303000	280	350	480	690	900	1100	1610			Y.P.	26500
91	4XD1	233200	270	400	530	640	720	1070	1230			Y.P.	24000
92	4XE1	258300	400	630	870	1080	1260	1440	1450			Y.P.	24600
93	4XF1	217100	250	400	540	650	730	800	1230			Y.P.	17700
94	4E5	98200											
95	4E25	74500											
96	4F5	105500											
97	4G5	106300											
98	4G25	64000											
99	4X5	148300											
100	4S1	176600							880	1030			21000
101	4T1	172200							860	1100			15000
102	4J1-2	177800							940				14500
103	4J11	155000							970				14000
104	4J21	150000							820				14000
105	4J31	74000							460				5000
106	4J3	129000							470				11500
107	4J6	226500							630				18000
108	4J26	201000							590				15500
109	4J8	233000							460				14000
110	4YJ8	194300							390				23000
111	4J28	238000							470				15000
112	4AG1-2	71300											
113	4AG21	51800											
114	4AG9	90800											

[A] Based on stresses in diag. web reinf. [F] Based on total web thickness, b'. [H] Based on b' minus diameter vertical web bars. [K] Based on b' minus diameter horizontal web bars.

TABLE 9.—*Deflections at varying shearing stresses and tensile stresses at crack widths of 0.01 and 0.02 inch*

NOTE: Stresses are given in pounds per sq. in.

Reference No.	Beam No.	Deflection at uncorrected shearing stress of:							Tensile stress in web reinf. when av. width of 5 largest cracks was:	
		200	400	600	800	1000	1200	1600	.01 in.	.02 in.
1	4A1-2	.004	.025	.049	.077	.109	.139		20600	
2	4B1-2	.010	.032	.058	.082	.115	.152		16000	
3	4C1-2	011	.031	.053	.079	.108	.137		20800	
4	4D1-2	.006	.027	.056	.081	.108	.145		21100	35400
5	4E1-2	.011	.034	.060	.085	.116	.147		22900	36300
6	4YE1	.017	.038	.058	.082	.109	.138	.265	12700	25800
7	4A3	.012	.038	.075	.109	.171			15700	25000
8	4B3	.011	.033	.066	.111	.190			16200	
9	4C3	.014	.038	.072	.105	.162			16700	28200
10	4D3	.011	.039	.072	.116	.170	.252		20700	30800
11	4E3	.012	.036	.068	.105	.155	.223		21200	33000
12	4E6-7	.017	.037	.075	.118	.171	.289		19400	37200
13	4E9	.020	.048	.098	.156	.265			20000	35200
14	4YE8	.018	.047	.095	.152	.236			6600	27000
15	4E11-12	.011	.031	.054	.082	.110	.143		16800	31200
16	4E21-2	.017	.038	.067	.097	.135			14900	
17	4E26	.019	.053	.088	.145				15500	27800
18	4E28-9	.014	.043	.096	.166				10600	29600
19	4E31	.020	.048	.081	.168				9300	12500
20	4M1-2	.012	.034	.060	.091	.132	.180		20100	
21	4YZ1	.000	.016	.035	.054	.073	.094	.145	21000	53500
22	4F1-2	.011	.030	.054	.074	.111	.143	.217	23700	44200
23	4YF1	.000	.034	.060	.087	.114	.146	.227	20500	44600
24	4F4	.013	.034	.052	.070	.093	.117	.178	21200	35000
25	4YF8	.013	.047	.105	.162				10500	31000
26	4F11-12	.011	.033	.053	.078	.110	.150		11700	31800
27	4F21-2	.014	.037	.061	.089	.183	.228		18900	33300
28	4F28	.019	.047	.094	.152				17000	27500
29	4F31	.018	.052	.130					13600	
30	4N1	.010	.029	.056	.085	.114	.147		37000	57000
31	4L1	.013	.034	.056	.079	.114	.166		31200	
32	4P1	.020	.045	.072	.100	.140	.195		38600	
33	4R1	.021	.043	.064	.091	.160			30200	
34	4G1-2	.020	.046	.071	.096	.126	.164		32700	53100
35	4YG1	.018	.038	.075	.114	.152	.198		27800	45700
36	4L3	.015	.034	.072	.112	.154	.212		32100	52200
37	4R3	.019	.040	.071	.102	.155			26500	45800
38	4G3	.020	.042	.082	.121	.164	.257		26200	46200
39	4G4	.010	.029	.047	.067	.086	.110	.170	23000	55000
40	4G6	.020	.048	.105	.155	.222			30000	50000
41	4G8	.009	.046	.104	.187				11800	25000
42	4YG8	.019	.058	.117	.193				3500	29600
43	4GA1	.020	.088	.178					8800	25200
44	4G11-12	.008	.031	.057	.084	.112	.148		26700	49700
45	4G21-2	.008	.029	.059	.094	.166			24000	39400
46	4G26-7	.009	.046	.083	.129	.226			11500	37600
47	4G28-9	.016	.048	.101	.200				4100	14300
48	4GA21	.020	.096	.185					18000	35300
49	4G31	.026	.061	.104	.203				26500	
50	4GB1	.003	.010	.177						29000
51	4K1	.000	.022	.052	.082	.124	.164		42300	60000
52	4H1-2	.013	.044	.076	.110	.163	.274		23700	47000
53	4H11-12	.013	.032	.059	.090	.125	.187		27300	53000
54	4H21-2	.015	.038	.073	.108	.152	.266		30800	56500
55	4H31	.025	.062						17800	31500
56	4BA21-2	.028	.087	.159	.288				31700	50500
57	4BB21-2	.031	.092	.180	.360				21700	31000
58	4BC21-2	.027	.068	.133	.287				17100	25600
59	4BD21	.033	.104	.225					8000	15900
60	4BD22	.027	.079	.165	.282				11800	16900
61	4BE21	.135							19000	37500
62	4CA1-2	.006	.018	.038	.058	.081	.104		55900[A]	
63	4CB1-2	.008	.024	.043	.064	.085	.108	.163	52400[A]	66000[A]

[A] Stresses in diagonal web reinforcement.

TABLE 10.—*Crack widths at varying shearing stresses and shearing stress at crack widths of 0.01 and 0.02 inch*

NOTE: Stresses are given in pounds per sq. in.

Reference No.	Beam No.	Average width of 5 largest cracks at uncorrected shearing stress of:							Shearing stress when av. width of 5 largest cracks was:	
		200	400	600	800	1000	1200	1600	.01 in.	.02 in.
1	4A1-2	.0000	.0028	.0063	.0089	.0116	.0145		890	
2	4B1-2	.0001	.0026	.0052	.0079	.0112	.0148		930	
3	4C1-2	.0001	.0024	.0050	.0075	.0101	.0134		990	
4	4D1-2	.0003	.0025	.0057	.0085	.0117	.0153		970	1600
5	4E1-2	.0000	.0028	.0062	.0088	.0118	.0155		870	1410
6	4YE1	.0022	.0077	.0110	.0136	.0162	.0188	.0286	520	1290
7	4A3	.0000	.0047	.0091	.0135	.0188			640	1030
8	4B3	.0000	.0042	.0090	.0130	.0168			650	
9	4C3	.0000	.0066	.0110	.0152	.0214			550	980
10	4D3	.0004	.0053	.0091	.0155	.0228	.0306		590	940
11	4E3	.0015	.0067	.0122	.0156	.0230	.0290		520	890
12	4E6-7	.0001	.0057	.0121	.0180	.0240	.0328		540	870
13	4E9	.0000	.0088	.0210	.0256	.0289			420	590
14	4YE8	.0000	.0090	.0156	.0244	.0372			410	700
15	4E11-12	.0013	.0063	.0087	.0114	.0139	.0172		690	1350
16	4E21-2	.0015	.0051	.0078	.0110	.0148			770	
17	4E26	.0017	.0099	.0182	.0264				410	650
18	4E28-9	.0000	.0073	.0176	.0247				450	710
19	4E31	.0016	.0068	.0094	.0166				560	860
20	4M1-2	.0000	.0035	.0062	.0093	.0130	.0174		850	
21	4YZ1	.0012	.0047	.0080	.0103	.0126	.0149	.0193	770	1670
22	4F1-2	.0002	.0053	.0087	.0122	.0140	.0160	.0242	670	1430
23	4YF1	.0000	.0080	.0099	.0116	.0134	.0151	.0219	610	1510
24	4F4	.0000	.0020	.0053	.0081	.0110	.0136	.0214	910	1540
25	4YF8	.0000	.0068	.0178	.0286				450	640
26	4F11-12	.0011	.0058	.0101	.0133	.0166	.0198		600	1220
27	4F21-2	.0009	.0045	.0087	.0136	.0194	.0255		660	1010
28	4F28	.0000	.0092	.0263	.0348				420	560
29	4F31	.0024	.0054	.0100					600	
30	4N1	.0000	.0033	.0067	.0087	.0117	.0154		880	1400
31	4L1	.0000	.0028	.0068	.0097	.0129	.0158		810	
32	4P1	.0007	.0023	.0045	.0078	.0107	.0130		940	
33	4R1	.0008	.0036	.0064	.0093	.0150			850	1090
34	4G1-2	.0006	.0050	.0088	.0141	.0179	.0227		620	1120
35	4YG1	.0010	.0069	.0099	.0148	.0202	.0253		600	990
36	4L3	.0000	.0045	.0091	.0138	.0203	.0306		640	990
37	4R3	.0000	.0045	.0109	.0159	.0236			570	910
38	4G3	.0012	.0083	.0153	.0225	.0296			450	730
39	4G4	.0006	.0064	.0090	.0110	.0146	.0166	.0194	700	1740
40	4G6	.0000	.0080	.0185	.0283				430	630
41	4G8	.0000	.0138	.0363					380	450
42	4YG8	.0000	.0084	.0270					420	550
43	4GA1	.0032	.0167	.0282					290	460
44	4G11-12	.0013	.0061	.0092	.0120	.0150	.0200		670	1190
45	4G21-2	.0005	.0054	.0096	.0140	.0189			620	1040
46	4G26-7	.0008	.0056	.0178	.0271				420	650
47	4G28-9	.0000	.0200	.0357					310	400
48	4GA21	.0010	.0120	.0261					340	520
49	4G31	.0024	0054	.0084	.0140				690	
50	4GB1	.0000	.0118	.0205					260	440
51	4K1	.0000	.0039	0075	.0110	.0165	.0236		750	1090
52	4H1-2	.0002	.0086	.0162	.0236	.0321			440	710
53	4H11-12	0017	.0060	0123	.0180	.0257	.0311		510	840
54	4H21-2	.0014	.0074	.0136	.0185				480	860
55	4H31	.0033	.0127						340	500
56	4BA21-2	.0004	.0066	.0125	.0236				520	750
57	4BB21-2	.0026	.0103	.0226	.0404				400	580
58	4BC21-2	.0007	.0099	.0214	.0350				420	570
59	4BD21	.0024	.0165	.0305					310	450
60	4BD22	.0014	.0110	.0260					390	530
61	4BE21	.0000	.0168						180	210
62	4CA1-2	.0000	.0010	.0033	.0049	.0074	.0101	.0190	1230	
63	4CB1-2	.0002	.0014	.0027	.0041	.0055	.0071	.0102	1600	2050

TABLE 9.—*Deflections at varying shearing stresses and tensile stresses at crack widths of 0.01 and 0.02 inch*—Continued

NOTE: Stresses are given in pounds per sq. in.

Reference No.	Beam No.	Deflection at uncorrected shearing stress of:							Tensile stress in web reinf. when av. width of 5 largest cracks was:	
		200	400	600	800	1000	1200	1600	.01 in.	.02 in.
64	4CC1-2	.008	.018	.034	.048	.064	.084	.123	61500[A]	Y.P.[A]
65	4CD1-2	.007	.026	.043	.061	.080	.099		49700[A]	Y.P.[A]
66	4CE1-2	.005	.017	.036	.054	.074	.093		47500[A]	
67	4U1-2	.011	.024	.041	.053	.070	.088	.138		
68	4AC1-2	.011	.024	.036	.051	.069	.085	.126		
69	4W1-2	.010	.028	.042	.061	.085	.107	.199	52400	
70	4AB1-2	.008	.021	.029	.040	.056	.065	.168	58000	65000
71	4W21	.000	.015	.031	.050	.064	.082		43000	
72	4X1-2-8	.020	.041	.060	.089	.116	.147	.164	52000	
73	4AA1-2	.004	.023	.046	.072	.096	.124	.227	54500	Y.P.
74	4X3	.011	.027	.046	.075	.132	.230		6220	
75	4X4	.002	.009	.022	.034	.047	.061	.093		
76	4X6	.004	.010	.028	.062	.095	.133		25700	54700
77	4X9	.009	.037	.076	.133				24500	46000
78	4X11	.010	.029	.048	.072	.097	.122	.221	48400	
79	4X21-2	.010	.027	.046	.068	.086	.117		57300	
80	4X31-2	.012	.029	.048	.084				23500	
81	4Y1-2	.011	.032	.053	.075	.113	.170		48900	Y.P.
82	4Z1	.022	.038	.055	.080	.114	.150		60500	Y.P.
83	4Y21	.010	.029	.047	.075	.104	.152		54000	Y.P.
84	4AH1-2	.003	.016	.029	.041	.055	.071	.137		
85	4AK2	.008	.022	.041	.051	.063	.084			
86	4AK1	.014	.027	.041	.056	.074	.092	.148		
87	4XG1	.011	.033	.054	.077	.105	.137			
88	4XA1	.011	.028	.048	.076	.112	.203		Y.P.	
89	4XB1	.010	.029	.062	.085	.115	.163		66000	
90	4XC1	.015	.033	.052	.076	.105	.135		Y.P.	
91	4XD1	.013	.032	.054	.084	.115	.185		Y.P.	
92	4XE1	.010	.020	.051	.070	.090	.123		60000	
93	4XF1	.009	.029	.049	.069	.107	.240		Y.P.	
		Deflection at total load of:						Max.[E]	Max.	
		18000	36000	54000	72000	108000	144000	Deflection	Load	
94	4E5	.040	.110	.190	.300			.730	98200	
95	4E25	.060	.140	.260	.520			.720	74500	
96	4F5	.040	.100	.130	.250	1.040		1.040	105500	
97	4G5	.040	.120	.190	.290	.790		.790	106300	
98	4G25	.050	.120	.230				.510	64000	
99	4X5	.020	.030	.040	.080	.150	.330	.400	148300	
100	4S1	.000	.010	.020	.040	.080	.120	.230	176600	
101	4T1	.010	.010	.020	.040	.080	.130	.240	172200	
102	4J1-2	.000	.010	.020	.050	.100	.150			
103	4J11	.000	.020	.040	.070	.130	.220	.290	155000	
104	4J21	.010	.010	.040	.070	.120	.210	.290	150000	
105	4J31	.000	.030	.070	.140			.200	74000	
		24000	48000	72000	96000	129000				
106	4J3	.010	.010	.030	. 30	.210				
		36000	72000	108000	144000	216000				
107	4J6	.010	.020	.030	.080	.170		.250	226500	
108	4J26	.000	.010	.060	.100			.280	201000	
		51000	102000	153000	204000					
109	4J8	.000	.010	.040	.110			.210	233000	
110	4YJ8	.010	.020	.050	.110			.210	194300	
111	4J28	.010	.020	.040	.110			.210	238000	
		18000	27000	36000	45000	54000	63000			
112	4AG1-2	.040		.130	.170	.230	.370	.470	70000	
113	4AG21	.050	.100	.200	.320			.540	51800	
		25000	51000	76000						
114	4AG9	.030	.100	.150				.340	90800	

[A] Stresses in diagonal web reinforcement. [E] Deflection at maximum load.

TABLE 10.—*Crack widths at varying shearing stresses and shearing stress at crack widths of 0.01 and 0.02 inch*—Continued

NOTE: Stresses are given in pounds per sq. in.

Reference No.	Beam No.	Average width of 5 largest cracks at uncorrected shearing stress of:							Shearing stress when av. width of 5 largest cracks was:	
		200	400	600	800	1000	1200	1600	.01 in.	.02 in.
64	4CCI-2	.0006	.0017	.0028	.0039	.0051	.0059	.0086	1890	2220
65	4CDI-2	.0003	0017	.0031	.0047	.0062	.0078	.0111	1480	2090
66	4CEI-2	.0000	.0009	.0041	.0062	.0081	.0120		1120	1390
67	4UI-2	.0001	.0007	.0018	.0028	.0035	.0045	.0069		
68	4ACI-2	.0001	.0006	.0016	.0027	.0036	.0043	.0060		
69	4WI-2	.0000	.0012	.0020	.0033	.0047	.0057	.0155	1550	
70	4ABI-2	.0001	.0012	.0024	.0034	.0046	.0054	.0134	1630	1750
71	4W21	.0000	.0009	.0019	.0035	.0053	.0071	.0110.	1500	
72	4X1-2-8	.0010	.0028	.0053	.0073	.0090	.0114	.0097	1200	
73	4AAI-2	.0004	.0023	.0036	.0052	.0070	.0085	.0169	1360	1630
74	4X3	.0000	.0015	.0042	.0070	.0100	.0162		1000	
75	4X4	.0000	.0004	.0011	.0018	.0026	.0035	.0051		
76	4X6	.0000	.0039	.0104	.0168	.0218			590	920
77	4X9	.0000	.0040	.0146	.0352				510	660
78	4X11	.0003	.0023	.0042	.0062	.0078	.0095	.0153	1250	
79	4X21-2	.0000	.0009	.0027	.0048	.0068	.0086	.0138	1420	
80	4X31-2	.0002	.0012	.0031	.0054					
81	4YI-2	.0001	.0033	.0069	.0098	.0141	.0242		790	1140
82	4ZI	.0000	.0028	.0056	.0096	.0172	.0310		820	1040
83	4Y21	.0000	.0023	.0051	.0080	.0117	.0198		910	1200
84	4AHI-2	.0000	.0003	.0011	.0020	.0029	.0035	.0047		
85	4AK2	.0000	.0012	.0013	.0017	.0026	.0035	.0053		
86	4AKI	.0000	.0016	.0029	.0041	.0058	.0067	.0080		
87	4XGI	.0000	.0007	.0019	.0032	.0046	.0061			
88	4XAI	.0002	.0016	.0035	.0053	.0070	.0087		1350	
89	4XBI	.0005	.0021	.0049	.0075	.0098	.0137		1020	
90	4XCI	.0006	.0018	.0030	.0043	.0060	.0080		1380	
91	4XDI		.0030	.0044	.0064	.0115	.0262		985	1115
92	4XEI		.0014	.0016	.0025	.0029	.0038		1435	
93	4XFI		.0027	.0059	.0095	.0136	.0250		825	1120
100	4S1	.0000	.0160	.0360						
101	4T1	.0005	.0117	.0400						
102	4J1-2	.0015	.0250	.0507	.0881					
103	4J11	.0050	.0248	.0575						
104	4J21	.0000	.0360	.0750	.1500					
105	4J31	.0120	.0550							
106	4J3	.0068	.0520							
107	4J6	.0000	.0420	.1150						
108	4J26	.0000	.0350							
109	4J8	.0000	.0700							
111	4J28	.0000	.0215							

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